Please adhere to the homework rules as given in the Syllabus.

1.   **Sample Spaces.** For each of the following experiments, (1) give the Sample Space using set notation, (2) determine whether each sample space is *discrete* or *continuous* and (3) give an example of an event. *note: describe your event in simple english, and also give it mathematically.*

   a) Experiment: Draw a card at random from a standard deck.
      Outcome: The **suit** of the chosen card.

   b) Experiment: A sixteen game season in the NFL.
      Outcome: The number of wins for the Arizona Cardinals.

   c) Experiment: An instagram account is selected at random.
      Outcome: The number of followers of this account.

   d) Experiment: Michael Phelps swims the 200m Butterfly.
      Outcome: The time it takes him to finish.

2. **Set Operations.** **a)** The difference (subtraction) operator $A - B$ reads "A minus B" is illustrated in the venn diagram below. Give an equivalent expression for $A - B$ using only unions, intersections and complements.
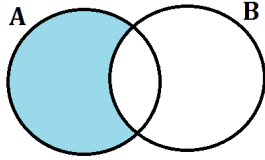
b) The symmetric difference operator $A \triangle B$ is illustrated in the venn diagram below. Note that this is similar to the logical idea of an *exclusive or*. Give an equivalent expression for $A \triangle B$ using only unions, intersections and complements.
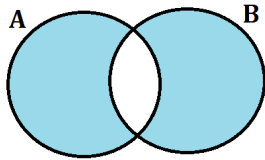
3. **Venn Diagrams!** For each part below, create a 3-set Venn Diagram (with a border representing S) and shade in the region corresponding to the given event.

a) $(A \cap B)^c$

b) $(A \cup B) \cap C$

c) $(A \cup B \cup C) \cap (A \cap B \cap C)^c$

4. Counting

a) Using a standard 52 card deck, how many ways can 13 cards be dealt to each of four different players?

b) Using a standard 52 card deck, how many ways can a 5 card poker hand be dealt? How many of these hands will be a flush? *a flush consists of five cards of the same suit.*

c) How many ways can two octopi shake hands if a hand shake consists of each octopus using a single tentacle?

d) How many ways can two octopi shake hands if a hand shake consists of each octopus using two tentacles? *for example, two humans can shake hands (using both hands) in just two ways: $(right \leftrightarrow right, left \leftrightarrow left)$ or $(left \leftrightarrow right, right \leftrightarrow left)$.*

5. **Rabbit island**. There are n rabbits living on an island, 10 of which have been captured and "tagged" in the past. Now, assume that researchers capture 20 rabbits, and find that 4 of them have been tagged.

a) What is the probability of this event occuring? *Note: your answer will be a function of n.*

b) Use R to plot this probability as a function of n, for values of n between 30 and 120. Based on your plot, what do you think is the *most likely* population size of rabbits on the island? *Note: Attach your plot to your homework.*

6. Professor Halfbrain has 10 books on mathematics, 8 books on chemistry and 5 books on astrology (he's a Gemini). He is packing for vacation, and hastily throws 7 books into his suitcase. What is the probability that he has selected at least 2 books from each subject?

7. A bit is either a 0 or a 1. A *byte* is a sequence of 8 bits and a *nibble* is a sequence of 4 bits.

a) How many different nibbles are possible? Write out the sample space.

b) What is the probability that a randomly generated nibble has two adjacent 1's?

8. **Challenge Problem.** *Ceci n'est pas une pipe.* Professor Halfbrain carries 2 matchboxes - 1 in his left pocket and 1 in his right pocket - at all times. Every time he smokes his pipe, he pulls a single match from one of the two boxes with equal probability. Assume that both matchboxes initially contained n matches.

At the moment that Professor Halfbrain discovers that one of his matchboxes is empty, what is the probability that there are exactly k matches remaining in the other box? $(k = 0, 1, 2, \cdots N)$.