STAT 345 Fall 2018
Homework 2 - Probability Name: ________________________________

Please adhere to the homework rules as given in the Syllabus.

1. **Probability Rules.** Fred is about to start watching "Game of Thrones" and "Westworld". The probability that Fred likes "Game of Thrones" is 0.58. The probability that Fred likes "Westworld" is 0.52. The probability that Fred likes both shows is 0.4. Define the events

$$A = \{\text{Fred likes Game of Thrones}\} \qquad B = \{\text{Fred likes Westworld}\}$$

a) Are A and B independent? Are they disjoint? Justify your answers mathematically.

b) Find the probability that Fred likes at least one of the two shows.

c) Find the probability that Fred doesn't like Westworld and he doesn't like Game of Thrones.

d) Find the probability that Fred likes Westworld, given that he doesn't like Game of Thrones.

2. **Conditional Probability.** A standard deck of 52 cards consists of 13 different ranks in each of 4 suits. A face card is any card having the rank J, Q or K. The suits "hearts" and "diamonds" are red, and the suits "spades" and "clubs" are black. **Use the formula for conditional probability to answer all the following questions.**

Professor Halfbrain draws a single card at random from the deck.

a) What is the probability that the cards suit is a heart, given that its color is red.

b) What is the probability that the card is red, given that it is a heart?

c) What is the probability that the cards rank is a J, given that its suit is a heart?

d) What is the probability that the cards rank is a J given that it is a face card?

e) What is the probability that the cards rank is an J given that it is not a red face card.

3. Let $A_1, A_2, \cdots A_n$ be independent events, such that $P(A_i) = \alpha$ for all $i = 1, 2, \cdots n$.

a) Find $P(A_1 \cup A_2 \cup A_3 \cup \cdots \cup A_n)$. *Hint: First look at the complement of this event, and then use DeMorgan's Law.*

b) How large does n need to be before the probability that $A_1 \cup A_2 \cup A_3 \cup \cdots \cup A_n$ exceeds 0.9? Your answer will be in terms of α. *Hint: Look at the inequality* $P(A_1 \cup \cdots \cup A_n) \geq 0.9$ *and solve for* n. *Don't forget that* $\log(x)$ *is negative when* $0 < x < 1$.

4. **Infinite Monkey Theorem.**

a) A monkey types each of the 26 letters on a typewriter (A,B,C,D,...,Y,Z) exactly once, to construct a 26 character "string" of letters. How many different strings are possible?

b) What is the probability that this 26 letter string contains the word "MONKEY"?
Hint: The first "task" is to choose the location, in the string, of the letter "M".

c) How many independent monkey typists would be required, so that the probability of the word "MONKEY" appearing at least once exceeds 0.9? *Hint: Combine your answer to part b) with your answer from Question 3.*

5. **Schrodingers Cat.** *I apologize in advance for this morbid problem.* Schrodinger asks his neighbor to feed his elderly cat while he is gone on vacation. If the neighbor remembers, the cat will die with probability 0.1. If the neighbor forgets, the cat will die with probability 0.8. Schrodinger is 90% sure that the neighbor will remember to feed the cat.

a) What is the probability that Schrodingers cat will be alive when he returns from vacation (ignoring quantum mechanics).

b) If the cat is dead when Schrodinger returns, what is the probability that his neighbor forgot to feed it?

6. In the Town of Somewhere it is either rainy (R), foggy (F) or sunny (S) with probabilities 0.3, 0.2 and 0.5 respectively.

- On days that it is rainy, Fred carries his umbrella 98% of the time.
- On days that it is foggy, Fred carries his umbrella half of the time.
- On days that it is sunny, Fred carries his umbrella 10% of the time.

This morning, I saw Fred carrying his umbrella. What is the probability that it is sunny?

7. Challenge Problem. *The Dating Problem.* Alf is ready to find a wife and settle down, so naturally, he turns to Tinder. Alf has N matches, each of which can be ranked from 1 to N. His goal is to find the best possible match with high probability. He decides to use the following probabilistic dating scheme.

- He will date each of his matches sequentially (in a completely random order). At the end of the date, he will make an *immediate* and *irrevocable* decision on whether or not to propose. (Yeah, Alf moves fast).
- For some integer $x > 1$, he will automatically reject the first $x - 1$ matches. He will then propose to the i^{th} match only if i^{th} date was better than all of the previous $i - 1$ dates.

a) Let $x = 2$ and find the probability that Alf finds his soulmate, i.e. propses to the best possible match. *Hint: Let A be the event that Alf proposes to the best match, and let B_i be the event that the i^{th} date is the best one. Use Law of Total Probability.*

b) Generalize your answer for any value of x less than N.

c) When N is large, the following identity holds (approximately).

$$\frac{x-1}{N}\sum_{i=x}^{N}\frac{1}{i-1} \approx -\frac{x}{N}\ln(x/N)$$

Using this approximation, show (using calculus) that the choice $x = N/e$ maximizes the probability of hiring the best candidate. If Alf has $N = 100$ matches, how many dates should he go on before he starts thinking about proposing? What is the probability that he finds his soulmate?