STAT 345 Spring 2018
Homework 4 - Discrete Probability Distributions Name: ______________________

Please adhere to the homework rules as given in the Syllabus.

1. Coin Flipping. Timothy and Jimothy are playing a betting game. Timothy will flip a fair coin until it lands Heads. If X is the total number of flips, then Jimothy has to pay Timothy $g(X)$ dollars, where $g(X)$ is defined as

$$g(X) = \begin{cases} -5, & X = 1 \\ 0, & X = 2, 3, 4 \\ M, & X \geq 5 \end{cases}$$

Using a computer, plot $E(g(X))$ as a function of M. For what value of M is the game "fair" in the sense that $E(g(X)) = 0$?

2. Professor Halfbrain has just created a fair five sided die, with sides numbered 1 through 5. This die is rolled once, and we let X be the number facing up.

a) Determine the expected value and the variance of X.

Professor Halfbrain constructs two more of these fair five sided die, but he chooses to number these 2 through 6 and 3 through 7 respectively.

b) If the Professor throws all 3 dice, what is the probability that the sum of their numbers is equal to 7? *Hint: Approach this as a counting problem.*

c) All three die are placed inside of a bag. The Professor reaches into the bag, grabs a die at random and tosses it. Let Y be the number facing up. Find $P(Y < 3)$. *Hint: Use Law of Total Probability.*

3. JeBron Lames is a basketball player who makes his free throws 70% of the time. Whether or not he makes a free throw is independent of his previous shots. Suppose JeBron shoots 10 freethrows, and let X be the number of free throws that he makes.

a) Identify the distribution of X, and write out it's PMF.

b) Determine the expected value and the variance of X.

c) Find $P(X = 5)$ and $P(X > 2)$.

4. Legendary STAT 345 Instructor Zach Stuart plans to swipe right until he gets a match. The probability of a match is (sadly) quite low at 4%. Let X be the number of times Zach will swipe right.

a) Identify the distribution of X, and write out it's PMF **and** it's CDF.

b) Determine the expected value and the variance of X.

c) What is the probability that Zach get's a match before he runs out of swipes (100 swipes per day).

5. Walter White and Jesse Pinkman run an operation where they cook blue(berry) muffins. Let X be the number of blueberries found in one of their muffins and assume that X follows a Poisson distribution with mean λ.

a) If there is a 2% chance that a muffin contains 0 blueberries, what is the value of λ?

b) What is the probability that there are at least 3 blueberries in a muffin? *Derive this probability by hand, using the PMF.*

c) What is the probability that there are at least 10 blueberries in a muffin? . *Write down the steps, but use R to get a final answer..*

d) Tuco buys a dozen (12) muffins, what is the probability that he gets at least 40 blueberries in total? *Write down your steps, but use R to get a final answer.*

6. Challenge Problem: Refer to the Bayes Theorem example we did in class, where Alf is being tested for a rare disease. Recall that, even if Alf tests positive the probability he has the disease is fairly small. Therefore Alf may need to take n tests instead of just one. If we let X be the number of times Alf tests positive, we are dealing with a Binomial distribution where the probability of success is conditional on whether or not Alf has the disease. Let $D = \{\text{Alf has the disease}\}$ and assume $P(D) = 0.00001$.

$$X|D \sim Binom(n, 0.99)$$
$$X|D^c \sim Binom(n, 0.05)$$

Now, assume that Alf tests positive for k tests ($k = 0, 1, \cdots n$). We can use Bayes Theorem as follows,

$$P(D|X = k) = \frac{P(X = k|D)P(D)}{P(X = k)}$$

Where $P(X = k) = P(X = k|D)P(D) + P(X = k|D^c)P(D^c)$ by Law of Total Probability.

a) Compute $P(X = k|D)$ and $P(X = k|D^c)$ using the appropriate Binomial PMF.

b) Use Bayes Theorem to give an explicit expression for $P(D|X = k)$ in terms of n and k.

c) If Alf takes $n = 4$ tests and tests positive $k = 3$ times, what is the probability he has the disease?

d) How many consecutive tests does Alf need to test positive for, before the probability he has the disease is at least 0.99?