STAT 345 Spring 2018
Homework 5 - Continuous Random Variables

Name: ______________________

Please adhere to the homework rules as given in the Syllabus.

1. Trapezoidal Distribution. Consider the following probability density function.

$$f(x) = \begin{cases} \frac{x+1}{5}, & -1 \leq x < 0 \\ \frac{1}{5}, & 0 \leq x < 4 \\ \frac{5-x}{5}, & 4 \leq x \leq 5 \\ 0, & otherwise \end{cases}$$

a) Sketch the PDF.

b) Show that the area under the curve is equal to 1 using geometry.

c) Find $P(X < 3)$ using geometry.

2. Let X have the following PDF, for $\theta > 0$.

$$f(x) = \begin{cases} \frac{c}{\theta^2}(\theta - x), & 0 < x < \theta \\ 0, & otherwise \end{cases}$$

a) Find the value of c which makes $f(x)$ a valid PDF. Sketch the PDF (or plot it using R).

b) Find the mean (i.e expected value) and standard deviation of X. *Your answers will be a function of* θ.

c) Find the CDF of X. Also sketch the CDF (or plot it using R).

d) The median of a RV, denoted η, is the value which is exceeded exactly half of the time. That is, η must satisfy the equation: $F(\eta) = 0.5$. Find the median of X.

e) Let $\theta = 2$ for the remainder of this question. Find the probability of each of the following events:

$$A = \{X > \sqrt{2}\} \qquad B = \{0.5 < X < 1.5\} \qquad C = \{X < 0.5 \ \cup \ X > 1.2\}$$

3. Let X be the height of a randomly selected UNM male student. Assume that X is normally distributed with mean $\mu = 70$ and standard deviation $\sigma = 3$.

a) Find the probability that a randomly selected UNM male student is taller than legendary rapper Lonzo Ball (6 ft 6 in).

b) Find the probability that a randomly selected UNM male student is between 5 feet and 6 feet tall.

c) If exactly 25% of UNM males are taller than Timothy, how tall is Timothy?

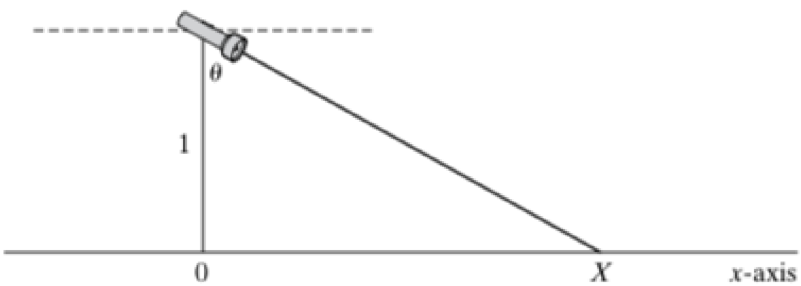
4. **The Log-normal Distribution.** In atmospheric science, the Log-normal distribution is often used to characterize particle size distributions. In one study, the distribution of silicone nanoparticle size (in nm) was found to be approximately Log-normal with Log-mean $\theta = 3.91$ and Log-sd $\omega = 0.47$. Suppose the desired range of particle sizes is $(40nm, 110nm)$. What percentage of silicone nanoparticles do you expect to fall within this range?

5. **Normal Approximations.**

a) Suppose that the number of cars that drive past Lomas on the I-25 between 5pm and 6pm on a Wednesday can be modeled as a Poisson random variable with mean $\lambda = 2100$. Use the Normal approximation to the Poisson distribution to determine the probability that less than 2000 cars drive past Lomas on the I-25 between 5pm and 6pm this wednesday.

b) 37% of cars in Albuquerque are white. Last wednesday, 2200 cars drove past Lomas on the I-25 between 5pm and 6pm. Use the Normal approximation to the Binomial distribution to determine the probability that more than 800 of these cars were white.

6. **Challenge Problem** *The Cauchy Distribution.* Suppose that a flashlight is spun around its center, which is located 1 unit away from the x-axis. (See Figure below). Consider the point X at which the beam intersects the x-axis when the flashlight has stopped spinning. (If the beam is not touching the x-axis at all, repeat the experiment.) In this problem, we will show that X has a Cauchy distribution (equivalent to a t-distribution with only 1 degree of freedom).



a) The angle θ (shown in picture) is also a random variable, and it is reasonable to assume that $\theta \sim Unif(-\pi/2, \pi/2)$. Write down the CDF of θ.

b) Determine (using Trig) the relationship between X and θ. Use this relationship, and your answer in a), to find $F(x)$, the CDF of X. What is the median of X? Does this make sense?

c) Find $f(x)$, the PDF of X.

d) An interesting feature of the Cauchy distribution, is that the expected value is *undefined* (even though the median is perfectly reasonable). Show that this is true by finding

$$I(a, b) = \int_a^b x f(x) dx$$

and showing that

$$\lim_{a \to -\infty} \lim_{b \to \infty} I(a, b)$$

does not equal

$$\lim_{b \to -\infty} \lim_{a \to \infty} I(a, b)$$