Discrete distribution

$X = \{x_1, x_2, \dots x_n\}$ – Sample space

X	x_1	x_2	x_3	$\dots$	x_n
$f(x)$	$f(x_1)$	$f(x_2)$	$f(x_3)$		$f(x_n)$

↳ Probability mass function

1) $x_1 + x_2 + \dots x_n \neq 1$

2) $f(x_1) + f(x_2) + \dots f(x_n) = 1$ ✓

3) $x_1 f(x_1) + x_2 f(x_2) + \dots x_n f(x_n) \neq 1$

4) $(x_1-\mu)^2 f(x_1) + (x_2-\mu)^2 f(x_2) + \dots (x_n-\mu)^2 f(x_n) \neq 1$

Experiment: flip a fair coin

X	1	0
$f(x)$	0.5	0.5

Mean (μ) : $\underline{\sum_{i=1}^{n} x_i f(x_i)}$

Ex $\mu = 0 \cdot 0.5 + 1 \cdot 0.5 = 0.5$

Variance (σ^2) : $\underline{\sum_{i=1}^{n} (x_i - \mu)^2 f(x_i)}$ ✓

mean

or

$$\sum_{i=1}^{n} x_i^2 f(x_i)$$

Ex $\sigma^2 = (0 - 0.5)^2 \cdot 0.5 + (1 - 0.5)^2 \cdot 0.5$

$= 0.25$

$$\mu = E(\underline{X}) = \sum_{i=1}^{n} \underline{x_i} f(x_i)$$

$$\sigma^2 = E(\underline{X - \mu})^2 = \sum_{i=1}^{n} (x_i - \mu)^2 f(x_i)$$

$$E(X^2) = \sum_{i=1}^{n} x_i^2 f(x_i)$$

$$E(g(x)) = \sum_{i=1}^{n} g(x_i) f(x_i)$$

$$E(X^2) = \sum_{i=1}^{n} x_i^2 f(x_i)$$

Ex. $E(X^2) = 0^2 \cdot 0.5 + 1^2 \cdot 0.5$
$= 0.5$

$$E(X^3) =$$

$$E(X-\mu)^2 =$$

Moment generating function:

$$M(t) = E(\underline{e^{tX}}) = \sum_{i=1}^{n} e^{tx_i} f(x_i)$$

t : input of the function

$$M(0) = \sum_{i=1}^{n} e^{0} \cdot f(x_i) = 1$$

$$M(1) = \sum_{i=1}^{n} e^{x_i} f(x_i)$$

$M'(t)$ evaluated at $0 \triangleq M'(0) = \overset{=\mu}{E(X)}$

$M''(t)$ evaluated at $0 \triangleq M''(0) = E(X^2)$

$M'''(t)$ evaluated at $0 \triangleq M'''(0) = E(X^3)$

x	0	1
f(x)	0.5	0.5

$\mu = 0.5$

$\sigma^2 = 0.25$

$E(x^2) = 0^2 \cdot 0.5 + 1^2 \cdot 0.5 = 0.5$

$$M(t) = E(e^{tx}) = \sum_{i=1}^{n} e^{tx_i} f(x_i)$$

$$= e^{t \cdot 0} \cdot 0.5 + e^{t \cdot 1} \cdot 0.5$$

$$= 0.5 + e^t \cdot 0.5$$

$(e^t)' = e^t$

$M'(t) = 0.5e^t$; $M'(0) = 0.5$

$M''(t) = 0.5e^t$; $M''(0) = 0.5$

$$\sigma^2 = M''(0) - [M'(0)]^2 \quad \left(E(x^2) - \mu^2\right)$$