Exercise set 1 for chapter 2

1 Probability

Problem 1. A computer system uses passwords that are six characters, and each character is one of the 26 letters (a-z) or 10 integers (0-9). Uppercase letters are not used. Let A denote the event that a password begins with a vowel (either a, e, i, o, or u), and let B denote the event that a password ends with an even number (either 0, 2, 4, 6, or 8). Suppose a hacker selects a password at random. Determine the following probabilities:

(a) $P(A)$

(b) $P(B)$

(c) $P(A \cap B)$

(d) $P(A \cup B)$

a) $P(A) = \dfrac{\binom{5}{1} \cdot 36^5}{36^6} = \dfrac{5}{36}$

b) $P(B) = \dfrac{5}{36}$

c) Since the last character choosing an even number is independent of the first character choosing a vowel,

$P(A \cap B) = P(A) \cdot P(B) = \dfrac{25}{36^2} = 0.0192$

d) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.258$

2 Conditional probabilities

Problem 2. Heart failures are due to either natural occurrences (87%) or outside factors (13%). Outside factors are related to induced substances (73%) or foreign objects (27%).

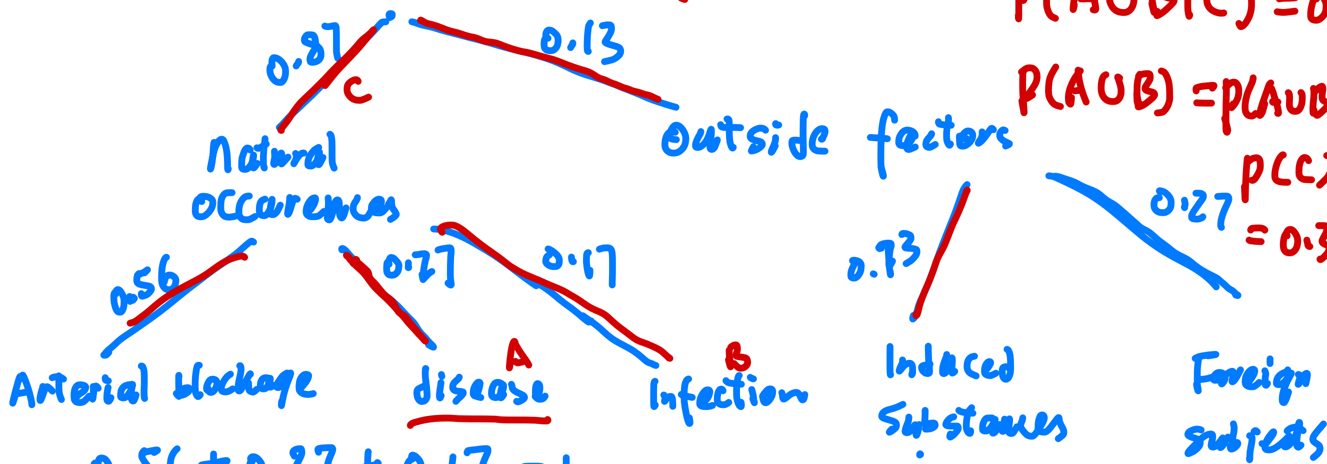
Natural occurrences are caused by arterial blockage (56%), disease (27%), and infection (e.g., staph infection) (17%).

(a) Determine the probability that a failure is due to an induced substance.

(b) Determine the probability that a failure is due to disease or infection.

$P(A \cup B | C) = 0.44$

$P(A \cup B) = P(A \cup B | C)\, P(C) = 0.383$



$0.56 + 0.27 + 0.17 = 1$

$P(\text{induced Substance}) = 0.13 \times 0.73 = 0.0949$

$P(\text{disease}) = 0.27 \times 0.87 = 0.2349$

$P(\text{infection}) = 0.17 \times 0.87 = 0.1479$

$P(\text{induced substance} | \text{outside factors}) = 0.73$

$P(\text{disease or infection}) = 0.2349 + 0.1479 = 0.383$

Since the two events are mutually exclusive.

3 Independence

Problem 3. A Web ad can be designed from four different colors, three font types, five font sizes, three images, and five text phrases. A specific design is randomly generated by the Web server when you visit the site. Let A denote the event that the design color is red, and let B denote the event that the font size is not the smallest one. Are A and B independent

$P(A \cap B) = P(A)P(B)$ or $P(A|B) = P(A)$ or $P(B|A) = P(B)$

$$P(A) = \frac{3 \times 5 \times 3 \times 5}{4 \times 3 \times 5 \times 3 \times 5} = \frac{1}{4}$$

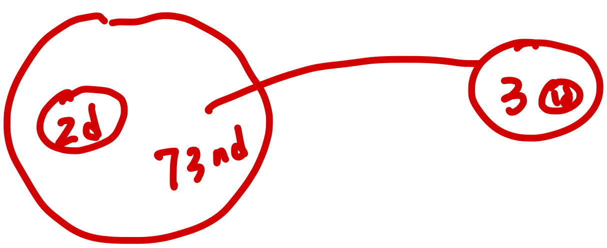
$$P(B) = \frac{4}{5}$$

$$P(A \cap B) = \frac{3 \times 4 \times 3 \times 5}{4 \times 3 \times 5 \times 3 \times 5} = \frac{1}{5}$$

$\rightarrow P(A \cap B) = P(A)P(B)$

Hence A and B are independent

Color is red and font size is not the smallest one events? Explain why or why not.



a) p(One defective)

$$= \frac{\text{\# of possible sample of 3 size that has exactly one defective}}{\text{\# of possible sample of 3 size}}$$

$$= \frac{2 \times \binom{73}{2}}{\binom{75}{3}} = \frac{2 \times \frac{73 \times 72}{2}}{\frac{75 \times 74 \times 73}{6}}$$

4 Combined knowledge

Problem 4. It is known that two defective cellular phones were erroneously sent to a shipping lot that now has a total of 75 phones. A sample of phones will be selected from the lot without replacement.

(a) If three phones are inspected, determine the probability that exactly one of the defective phones will be found.

step1: select the defective phone from the set of 2 defective phones

(b) If three phones are inspected, determine the probability that both defective phones will be found.

step2: select 2 nd from 73 nd

(c) If 73 phones are inspected, determine the probability that both defective phones will be found.

b) $$\frac{\binom{73}{1} \cdot \binom{2}{2}}{\binom{75}{3}}$$

c) $$\frac{\binom{73}{71} \cdot \binom{2}{2}}{\binom{75}{73}}$$

$$\binom{73}{71} = \binom{73}{2}$$