

Exercise set 2 for Chapter 3 and Chapter 4

Problem 1. A multiple-choice test contains 25 questions, each with four answers. Assume that a student just guesses on each question.

- (a) Let X denotes the number of questions the student answer correctly. What is the distribution of X ?
- (b) What is the probability that for between three and ten questions (including three and ten), the student answer correctly?
- (c) What is the expected number of questions the student answer correctly?

Problem 2. In a clinical study, volunteers are tested for a gene that has been found to increase the risk for a disease. The probability that a person carries the gene is 0.1. People are assumed to be independent with respect to the gene.

- (a) What is the probability that exactly four people need to be tested to detect one person with the gene?
- (b) What is the probability that four or more people need to be tested to detect one person with the gene?
- (c) What is the expected number of people to test to detect one person with the gene?

Problem 3. The number of telephone calls that arrive at a phone exchange is often modeled as a Poisson random variable. Assume that on the average there are 10 calls per hour. Determine the following probabilities:

- (a) 3 or fewer calls in one hour
- (b) 3 or fewer calls in half one hour

Problem 4. Patient response to a generic drug to control pain is scored on a 5-point scale where a 5 indicates complete relief. The distribution of scores is displayed below. Two patients, assumed to be independent, are each scored. What is the probability mass function of the total score?

Table 1: Probability distribution

x	1	2	3	4	5
Probability	0.05	0.1	0.2	0.25	0.4

Problem 5. The probability density function of the net weight in pounds of a packaged chemical herbicide is $f(x) = 2.0$ for $49.75 < x < 50.25$ pounds.

- (a) Determine the probability that a package weighs more than 50 pounds.
- (b) What is the cumulative distribution of the chemical net weight?
- (c) What is the 90% percentile of the chemical net weight?
- (d) What are the mean and variance for the distribution of the chemical net weight?

Problem 6. The weight of a running shoe is normally distributed with a mean of 12 ounces and a standard deviation of 0.5 ounce.

- (a) What is the probability that a shoe weighs more than 13 ounces?
- (b) Given that a shoe weighs more than 10 ounces, what is the probability that it weighs more than 13 ounces?
- (c) What must the standard deviation of weight be in order for the company to state that 99.9% of its shoes weighs less than 13 ounces?
- (d) If the standard deviation remains at 0.5 ounce, what must the mean weight be for the company to state that 99.9% of its shoes weighs less than 13 ounces?

Problem 7. If $X \sim N(\mu, \sigma^2)$, find the density for random variable $Y = e^X$. The distribution

of Y will be known as lognormal distribution.