# Exercise set for chapter 5-8

## 1 Chapter 5

**Problem 1.** Two random variables have a joint probability mass function in Table 1. Show that the correlation between $X$ and $Y$ is zero but $X$ and $Y$ are not independent.

Table 1: Joint probability mass function

| $x$ | -1 | 0 | 0 | 1 |
|---|---|---|---|---|
| $y$ | 0 | -1 | 1 | 0 |
| $f_{X,Y}(x,y)$ | 1/4 | 1/4 | 1/4 | 1/4 |

**Problem 2.** A random variable $X$ has the probability distribution $f_X(x) = x/18, 0 \leq x \leq 6$. Determine the following:

(a) The expected value of $Y$.

(b) The variance of $Y$.

(c) The cumulative distribution of $Y$.

**Problem 3.** $X$ and $Y$ are independent, normal random variables with $E(X) = 2$, $Var(X) = 5$, $E(Y) = 6$, $Var(Y) = 8$.

(a) $E(3X + 2Y)$.

(b) $Var(3X + 2Y)$.

(c) Note that the sum of two normal random variables is still a normal random variable. What is the distribution of $3X + 2Y$? Please specify its mean and variance.

## 2 Chapter 6

**Problem 4.** Consider the following two samples: Sample 1: 10, 9, 8, 7, 8, 6, 10, 6; Sample 2: 10, 6, 10, 6, 8, 10, 8, 6.

(a) Calculate the sample range for both samples. Would you conclude that both samples exhibit the same variability? Explain.

(b) Calculate the sample standard deviations for both samples. Do these quantities indicate that both samples have the same variability? Explain.

# 3 Chapter 7

**Problem 5.** Describe the population/distribution parameter that the following numerical summaries trying to estimate.

(a) Sample mean

(b) Sample variance

(c) Sample proportion

**Problem 6.** A random sample of size $n_1 = 16$ is selected from a normal population with a mean of 75 and a standard deviation of 8. A second random sample of size $n_2 = 9$ is taken from another normal population with mean 70 and standard deviation 12. Let $\bar{X}_1$ and $\bar{X}_2$ be the two sample means. Note the difference of two normal random variables is still a normal random variable. Find:

(a) The probability that $\bar{X}_1 - \bar{X}_2$ exceeds 4.

(b) The 95th percentile of the distribution for $\bar{X}_1 - \bar{X}_2$.

**Problem 7.** Suppose we have a random sample of size $2n$ from a population denoted by $X$, and $E(X) = \mu$, $Var(X) = \sigma^2$. Let $\bar{X}_1 = \frac{1}{2n}\sum_{i=1}^{2n} X_i$ and $\bar{X}_2 = \frac{1}{2}\sum_{i=1}^{n} X_i$ be two estimators of $\mu$. Which is the better estimator of $\mu$? Hint, compare the bias and the MSE of the two estimators.

# 4 Chapter 8

An article in the Journal of the American Statistical Association ["Illustration of Bayesian Inference in Normal Data Models Using Gibbs Sampling" (1990, Vol. 85, pp. 972?985)] measured the weight of 30 rats under experiment controls. Suppose that 12 were underweight rats.

(a) Calculate a 95% two-sided confidence interval on the true proportion of rats that would show underweight from the experiment.

(b) Using the point estimate of p obtained from the preliminary sample, what sample size is needed to be 95% confident that the error in estimating the true value of $p$ is less than 0.02?

(c) How large must the sample be if you wish to be at least 95% confident that the error in estimating p is less than 0.02, regardless of the true value of $p$?