Homework 2

Please complete the problems on a separate sheet of paper with your name at the top. Make sure to show your work and/or provide an explanation for each problem. Please be clear in your work. Partial credit will be given when merited. The total credit is 6 points.

Problem 1. If P(A) = 0.2, P(B) = 0.2, and $P(A \cap B) = 0.1$, determine the following probabilities and write down the steps. For example, $P(A|B) = P(A \cap B)/P(B) = 0.1/0.2 = 0.5$. Final answer only get half of the credit.

- (a) $P(A \cup B)$ (0.2 point)
- (b) $P(A' \cap B)$ (0.2 point)
- (c) $P(A \cap B')$ (0.2 point)
- (d) P(A|B') (0.2 point)
- (e) $P(A|A \cup B)$ (0.2 point)

Problem 2. Determine true or false of the following and explain why or cite the proper content from the note (each worth 0.25 point):

- (a) For a discrete sample space, the probability of an event is the sum of all of the probabilities of the outcomes associated with the event.
- (b) You are driving and come to a fork in the road. Event A is the event that you turn left. Event B is the event that you turn right. A and B are mutually exclusive.
- (c) P(A|B) + P(A'|B) = 1.
- (d) P(B) = P(B|A)P(B) + P(B|A')P(B).

Problem 3. A maintenance firm has gathered the following information regarding the failure mechanism for 100 air conditioning systems in the table below. If this is a representative sample of AC failure, find to 3 decimal places the probability

(a) That a failure involves a gas leak. (0.1 point)

- (b) That there is evidence of electrical failure given that there was a gas leak. (0.2 point)
- (c) That there is evidence of a gas leak given that there is evidence of electrical failure. (0.2 point)

| | | Evidence of Gas Leaks | |
|--------------------------------|-----|-----------------------|----|
| | | Yes | No |
| Evidence of electrical failure | Yes | 51 | 2 |
| | No | 32 | 15 |

Problem 4. The probability is 1% that an electrical connector that is kept dry fails during the warranty period of a portable computer. If the connector is ever wet, the probability of a failure during the warranty period is 4%. Suppose 90% of the connectors are kept dry and 10% are wet.

- (a) What proportion of connectors fail during the warranty period? (0.25 point)
- (b) Given that a connector fail during the warranty period, what is the probability that it is ever wet? (0.25 point)

Problem 5. A player of a video game is confronted with a series of four opponents and an 70% chance of defeating an opponent. Assume that the results from opponents are independent.

- (a) What is the probability that a player defeats all four opponents in a game? (0.25 point)
- (b) What is the probability that a player defeats at least two opponents in a game? (0.75 point)

Problem 6. A batch of 472 containers for frozen orange juice contains 4 that are defective. Let A and B denote the events that the first and second container selected is defective, respectively.

- (a) If the two are selected, at random, without replacement from the batch.
 - Obtain P(A), P(B). (0.5 point)
 - Are A and B independent events? Why? (0.5 point)
- (b) If the sampling were done with replacement,
 - Obtain P(A), P(B). (0.5 point)
 - Would A and B be independent? Why? (0.5 point)