## Homework 2

Please complete the problems on a separate sheet of paper with your name at the top. Make sure to show your work and/or provide an explanation for each problem. Please be clear in your work. Partial credit will be given when merited. The total credit is 6 points.

Problem 1. If $P(A)=0.2, P(B)=0.2$, and $P(A \cap B)=0.1$, determine the following probabilities and write down the steps. For example, $P(A \mid B)=P(A \cap B) / P(B)=0.1 / 0.2=$ 0.5 . Final answer only get half of the credit.
(a) $P(A \cup B)(0.2$ point $)$
(b) $P\left(A^{\prime} \cap B\right)(0.2$ point $)$
(c) $P\left(A \cap B^{\prime}\right)(0.2$ point $)$
(d) $P\left(A \mid B^{\prime}\right)(0.2$ point $)$
(e) $P(A \mid A \cup B)(0.2$ point $)$

Problem 2. Determine true or false of the following and explain why or cite the proper content from the note (each worth 0.25 point):
(a) For a discrete sample space, the probability of an event is the sum of all of the probabilities of the outcomes associated with the event.
(b) You are driving and come to a fork in the road. Event A is the event that you turn left. Event B is the event that you turn right. A and B are mutually exclusive.
(c) $P(A \mid B)+P\left(A^{\prime} \mid B\right)=1$.
(d) $P(B)=P(B \mid A) P(B)+P\left(B \mid A^{\prime}\right) P(B)$.

Problem 3. A maintenance firm has gathered the following information regarding the failure mechanism for 100 air conditioning systems in the table below. If this is a representative sample of AC failure, find to 3 decimal places the probability
(a) That a failure involves a gas leak. (0.1 point)
(b) That there is evidence of electrical failure given that there was a gas leak. (0.2 point)
(c) That there is evidence of a gas leak given that there is evidence of electrical failure. (0.2 point)

|  |  | Evidence of Gas Leaks |  |
| :--- | :--- | :--- | :--- |
|  |  | Yes | No |
| Evidence of electrical failure | Yes | 51 | 2 |
|  | No | 32 | 15 |

Problem 4. The probability is $1 \%$ that an electrical connector that is kept dry fails during the warranty period of a portable computer. If the connector is ever wet, the probability of a failure during the warranty period is $4 \%$. Suppose $90 \%$ of the connectors are kept dry and $10 \%$ are wet.
(a) What proportion of connectors fail during the warranty period? ( 0.25 point)
(b) Given that a connector fail during the warranty period, what is the probability that it is ever wet? ( 0.25 point)

Problem 5. A player of a video game is confronted with a series of four opponents and an $70 \%$ chance of defeating an opponent. Assume that the results from opponents are independent.
(a) What is the probability that a player defeats all four opponents in a game? ( 0.25 point)
(b) What is the probability that a player defeats at least two opponents in a game? (0.75 point)

Problem 6. A batch of 472 containers for frozen orange juice contains 4 that are defective. Let $A$ and $B$ denote the events that the first and second container selected is defective, respectively.
(a) If the two are selected, at random, without replacement from the batch.

- Obtain $P(A), P(B)$. (0.5 point)
- Are $A$ and $B$ independent events? Why? (0.5 point)
(b) If the sampling were done with replacement,
- Obtain $P(A), P(B)$. ( 0.5 point)
- Would $A$ and $B$ be independent? Why? (0.5 point)

