Homework 3

Please complete the problems on a separate sheet of paper with your name at the top. Make sure to show your work and/or provide an explanation for each problem. Please be clear in your work. Partial credit will be given when merited. Each problem worth 1 point. The total credit is 7 points.

Problem 1. Let X be a random variable with the following **probability mass function**:

$$f(x) = \frac{2x+1}{25}; x = 0, 1, 2, 3, 4$$

(a) Find $P(2 \leq X < 4)$ (0.25 point)

(b) Determine the cumulative distribution function of X based on the probability mass function given above. (0.25 point)

(c) Find the mean of X. (0.25 point)

(d) Find the variance of X. (0.25 point)

Problem 2. Suppose that a large batch of electrical fuses contain 5% defectives. Suppose a sample of 5 fuses are tested and let X be the number of defectives among the 5 fuses.

(a) Find the probability mass function of X. (0.2 point)

(b) Find the mean of X. (0.2 point)

(c) Find variance of X. (0.2 point)

(d) Find the probability for that less than 2 defective fuses are observed. (0.2 point)

(e) Find the probability for that greater than or equal to 2 defective fuses are observed. (0.2 point)

Problem 3. The following function is cumulative distribution function.

$$F(x) = \begin{cases} 0 & x < -15 \\ 0.15 & -15 \leq x < 25 \\ 0.70 & 25 \leq x < 45 \\ 1 & 45 \leq x \end{cases}$$

(a) $P(X \leq 45)$. (0.25 point)

(b) $P(35 \leq X \leq 55)$. (0.25 point)

(c) Find the probability mass function of X. (0.5 point)

Problem 4. Because not all airline passengers show up for their reserved seat, an airline sells 125 tickets for a flight that holds only 115 passengers. The probability that a passenger does not show up is 0.05, and the passengers behave independently.

(a) Define X as the number of passenger showing up among 125 passengers. Give the probability mass function of X. (0.5 point)

(b) What is the probability that every passenger who shows up can take the flight, i.e. $P(X \leq 115)$? (0.25 point)

(c) What is the expected number of passengers who will show up among 125 passengers, i.e. $E(X)$? (0.25 point)

Problem 5. Assume that each of your calls to a popular radio station has a probability of $p = 0.03$ of connecting, that is, of not obtaining a busy signal. Assume that your calls are independent.

(a) Define X as the number of calls you need to make until the first connect. Give the probability mass function of X. (0.5 point)

(b) What is the probability that your 1st call that connects is your 10th call, i.e. $P(X = 10)$? (0.25 point)

(c) What is the probability that it requires more than 5 calls for you to connect,, i.e. $P(X > 5)$? (0.25 point)

Problem 6. If X is a Poisson random variable from a Poisson process with parameter λ in a given continuous interval of length T, then the probability mass function is:

$$f(x) = \frac{e^{-\lambda T}(\lambda T)^x}{x!} \text{ where } x = 0, 1, 2, \ldots$$

(a) Show the moment generating function of X. (0.5 point)

(b) Prove that $E(X) = \lambda T$. (0.25 point)

(c) Prove that $Var(X) = \lambda T$. (0.25 point)

Problem 7. Suppose X is a discrete random variable with mean μ and variance σ^2. Let $Y = X + 1$.

(a) Derive $E(Y)$. (0.5 point)

(b) Derive $Var(Y)$. (0.5 point)