Homework 5

Please complete the problems on a separate sheet of paper with your name at the top. Make sure to show your work and/or provide an explanation for each problem. Please be clear in your work. Partial credit will be given when merited. The total credit is 7 points.

Problem 1. (2.5 points total) Random variable X and Y have the a joint probability mass function $f_{X,Y}(x, y) = c(x+y)$ over the a sample space $(X, Y) = \{(1, 1), (1, 2), (2, 1), (2, 2), (2, 4), (3, 2)\}$. (4 pts total)

(a) Determine the value c so that the joint probability mass function is legitimate. (0.5 point)

(b) Find the marginal probability mass function of X and Y respectively. (0.5 point)

(c) Find the mean of X using the marginal distribution of X. (0.5 point)

(d) Find the mean of Y using the marginal distribution of Y. (0.5 point)

(e) Find $E(XY)$. (0.5 point)

(f) Find the covariance of X and Y.(0.25 point)

(g) Find the variance of X using the marginal distribution of X. (0.5 point)

(h) Find the variance of Y using the marginal distribution of Y. (0.5 point)

(i) Find the correlation of X and Y. Are X and Y independent? Why? (0.25 point)

Problem 2. Suppose X_1 and X_2 are two random variables with $E(X_1) = 2$, $E(X_2) = 3$, $Var(X_1) = 1$, $Var(X_2) = 2$ and covariance $COV(X_1, X_2) = 1$.

(a) Find the mean of $X_1 + 3X_2$. (0.25 point)

(b) Find the variance of $X_1 + 3X_2$. (0.25 point)

Problem 3. Suppose $X_1, \ldots, X_{10}$ is a **simple random sample** from a Normal distribution $N(2, 3^2)$. Simple random sample means that $X_1, \ldots, X_{10}$ are independent random variables and they are identically distributed. Define $\bar{X} = \frac{1}{n}\sum_{i=1}^{10} X_i$ is the sample mean.

(a) What is $E(\bar{X})$? (0.25 point)

(b) What is $Var(\bar{X})$? (0.25 point)

(c) What is the distribution of $\bar{X}$? Please specify the name of the distribution, mean, and variance of the distribution. (0.25 point)

(d) What is the probability that the random sample mean falls in the interval $[1.5, 2.5]$? (0.25 point)

Problem 4. Suppose that a simple random sample (refer to problem 3 for what it means) of $n = 15$ observations is selected from a continuous uniform distribution over $[0, 1]$.

(a) What are the mean and variance of $\bar{X}$? (0.5 point)

(b) What is the approximate distribution of $\bar{X}$? Please specify the name of the distribution, mean, and variance of the distribution. (0.5 point)

(c) What is the approximate distribution of $\bar{X} - 7$? Please specify the name of the distribution, mean, and variance of the distribution. (0.5 point)