Homework 2

Problem 1:

a) $p(A \cup B) = p(A) + p(B) - p(A \cap B) = 0.2 + 0.2 - 0.1 = 0.3$

b) $p(A^c \cap B) = p(B) - p(A \cap B) = 0.2 - 0.1 = 0.1$

c) $p(A \cap B^c) = p(A) - p(A \cap B) = 0.2 - 0.1 = 0.1$

d) $p(A \mid B') = \dfrac{p(A \cap B')}{p(B')} = \dfrac{0.1}{1 - 0.2} = 0.125$

e) $p(A \mid A \cup B) = \dfrac{p(A)}{p(B)} = \cancel{1}$ $\dfrac{2}{3}$

problem 2

a) True. since $p(A) = \sum_{s_i \in A} p_i$ where p_i is the probability of s_i

b) True since A and B cannot happen at the same time.

c) True. $P(A \mid B) + p(A' \mid B) = \dfrac{p(A \cap B)}{p(B)} + \dfrac{p(A' \cap B)}{p(B)} = \dfrac{p(B)}{p(B)} = 1$

d) False since $p(B) = p(B \mid A')\, p(A') + p(B \mid A)\, p(A)$

Problem 3

a) $\frac{51+32}{100} = 0.83$

b) $p(\text{Electrical failure} \mid \text{Gas leak}) = \frac{51}{83} = 0.614$

c) $p(\text{Gas leak} \mid \text{Electrical failure}) = \frac{51}{53} = 0.962$

Problem 4.

a) $p(\text{fail}) = p(\text{fail} \mid \text{connector is kept dry})\, p(\text{connector is kept dry})$

$+ p(\text{fail} \mid \text{connector is ever wet})\, p(\text{connector is eve}$
$\text{wet})$

$= 0.01 \cdot 0.9 + 0.04 \cdot 0.1 = 0.013$

b) $p(\text{connector is ever wet} \mid \text{fail}) = \dfrac{p(\text{connector fail} \mid \text{connector is e wet}) \cdot p(\text{con is e we}}{p(\text{fail})}$

$= \dfrac{0.04 \cdot 0.1}{0.013} = 0.308$

Problem 5

a) $0.7^4 = 0.2401$ since all games are independent.

b) $\binom{4}{2}0.7^2 \cdot 0.3^2 + \binom{4}{3} \cdot 0.7^3 \cdot 0.3 + \binom{4}{4} \cdot 0.7^4$

$= 0.9163$

$\binom{4}{2}0.7^2 \cdot 0.3^2$: probability for exactly 2 wins

$\binom{4}{3} \cdot 0.7^3 \cdot 0.3$: probability for exactly 3 wins

$\binom{4}{4} \cdot 0.7^4$: probability for exactly 4 wins

Problem 6 :

a) $P(A) = \frac{4}{472}$

$P(B|A) = \frac{3}{471}$ $\quad P(B|A^c) = \frac{4}{471}$

$P(B) = P(B|A) \cdot P(A) + P(B|A^c)P(A^c)$

$= \frac{3}{471} \cdot \frac{4}{472} + \frac{4}{471} \cdot (1 - \frac{4}{472})$

$= \frac{1884}{471 \cdot 472} \approx 0.0096$

Since $P(B|A) \approx 0.00637 \neq P(B)$, A and B are not independent.

b) In this case

$$p(A) = \frac{4}{472}$$

$$p(B|A) = \frac{4}{472} \quad , \quad p(B|A^c) = \frac{4}{472}$$

$$p(B) = \frac{4}{472} \cdot \frac{4}{472} + \frac{4}{472} \cdot \left(1 - \frac{4}{472}\right) = \frac{4}{472}$$

Hence $p(B) = p(B|A)$

Therefore A and B are independent.