STAT 345 - HW 5

1. a) $f_{x,y}(x,y) = c(x+y)$

Sum of all pmf should be 1

$c(2+3+3+4+6+5) = 1$

$c = \frac{1}{23}$

b)

X \ Y	1	2	3	4	$f_X(x)$
1	2/23	3/23	0	0	5/23
2	3/23	4/23	0	6/23	13/23
3	0	5/23	0	0	5/23
$f_Y(y)$	5/23	12/23		6/23	

$$f_X(x) = \begin{cases} 5/23 & x=1 \\ 13/23 & x=2 \\ 5/23 & x=3 \\ 0 & \text{otherwise} \end{cases}$$

$$f_Y(y) = \begin{cases} 5/23 & y=1 \\ 12/23 & y=2 \\ 6/23 & y=4 \\ 0 & \text{otherwise} \end{cases}$$

c) $$\mu_X = 1\left(\frac{5}{23}\right) + 2\left(\frac{13}{23}\right) + 3\left(\frac{5}{23}\right)$$

$$= 2$$

d) $$\mu_Y = 1\left(\frac{5}{23}\right) + 2\left(\frac{12}{23}\right) + 4\left(\frac{6}{23}\right)$$

$$= 2.304$$

e) $$E(XY) = 1\cdot\frac{2}{23} + 2\cdot\frac{3}{23} + 2\cdot\frac{3}{23} + 4\cdot\frac{4}{23} + 8\cdot\frac{6}{23} + 6\cdot\frac{5}{23}$$

$$= 4.695$$

f) $Cov(X,Y) = E(XY) - \mu_X \mu_Y$

$= 4.695 - 2 \cdot 2.304$

$= 0.087$

g) $Var(X) = 1^2 \cdot \frac{5}{23} + 2^2 \cdot \frac{13}{23} + 3^2 \cdot \frac{5}{23} - (2)^2$

$= 0.435$

h) $Var(Y) = 1^2 \cdot \frac{5}{23} + 2^2 \cdot \frac{12}{23} + 4^2 \cdot \frac{6}{23} - (2.304)^2$

$= 1.168$

i) $Cor(X,Y) = \frac{\sigma_{XY}}{\sigma_X \sigma_Y} = \frac{0.087}{\sqrt{0.435 \cdot 1.168}}$

$= 0.123 \quad (0.121 - 0.123)$

$f_X(1) \, f_Y(1) = 0.0473 \neq f(1,1) = 0.087$

$\therefore$ X and Y, Not independent

2. a) $1 \cdot E(X_1) + 3 \cdot E(X_2)$

$= 1 \cdot (2) + 3 \cdot (3)$

$= 11$

b) $1^2 \cdot 1 + 3^2 \cdot 2 + 2(1 \cdot 3 \cdot 1)$

$= 25$

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3. $N(2, 3^2)$

$\bar{X} = \frac{1}{n} \sum_{i=1}^{10} X_i$

a) $E(\bar{X}) = \frac{1}{10}(10 \cdot 2)$

$= 2$

b) $\frac{\sigma^2}{n} = \frac{3^2}{10} = \frac{9}{10}$

c) It is a normal distribution
$\bar{X} \sim N\left(2, \frac{9}{10}\right)$ where $\mu = 2$, $\sigma = \frac{9}{10} = 0.9$

d) $P(1.5 < \bar{X} < 2.5)$

$P\left(\frac{1.5-2}{\sqrt{0.9}} < \frac{\bar{X}-2}{\sqrt{0.9}} < \frac{2.5-2}{\sqrt{0.9}}\right)$

$= P(-0.53 \leq z \leq 0.53) = P(z \leq 0.53) - P(z \leq 0.53)$

$= 0.7019 - 0.2981$

$= 0.4038$

4. a) $\mu = \frac{b+a}{2} = \frac{0+1}{2} = 0.5$

$\sigma^2 = \frac{(b-a)^2}{12} = \frac{(1-0)^2}{12} = \frac{1}{12}$

b) It is a normal distribution with

$\mu = 0.5$

$\sigma^2 = \frac{1/12}{15} = 0.00556$

c) $\bar{X} - 7$ Normal distribution with

$\mu = 0.5 - 7$
$= -6.5$

$\sigma^2 = Var(\bar{X} - 7) = var(\bar{X}) = 0.00556$