Introduction and Chapter 2

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Main topics

- Random experiments
  - Sample space, event
  - Counting techniques
- Probability
  - Probability and its axioms
  - Conditional probability and Bayes' rule
  - Random variables
- Distributions
  - Discrete distributions
  - Continuous distributions
  - Expectation, Variance, Moment generating function
- Descriptive Statistics
- Statistical inference
  - Point Estimation
  - Confidence interval
  - Hypothesis testing

Objectives of this class

Preparing you for mastering applied statistical methods in your fields

- Help you understand basic probability and statistics concepts and methods
- Help you apply the concepts and methods to practical problems
- Help you carry out basic statistical analysis and interpret

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- Good statistics offer critical guidance in producing trustworthy analyses and predictions.
- Potential analytical errors
  - Biased samples, overgeneralization
  - Causality vs correlation
  - Incorrect analysis
  - Violating the assumptions for an analysis

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- John Tukey: “The best thing about being a statistician is that you get to play in everyone else’s backyard.”

Our majors in this class

- Pure math, applied math
- Statistics
- Mechanical engineering, nuclear engineering, civil engineering
- Earth and planetary sciences
- Biology, nursing, psychology
- ...

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- Ask questions
- Seek help from me or tutor in a timely manner

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- Anyone has super power in flipping coins in this class? Experiment for each two of you:
  - One flips a quarter 10 times
  - One records the maximum number of heads that are consecutive–referred as “number of consecutive heads” below
  - For the team with the highest number, repeat the experiment by the same person

Random experiment

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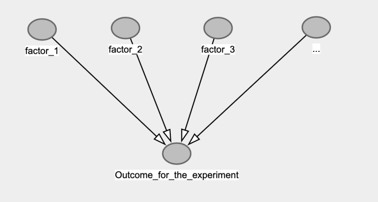
- Why there is variability in the number of heads even if experiment is repeated for the same team?

Random experiment

- Reason for the phenomenon of randomness in the results from repeated experiments: unmeasured factors cause the results to be different



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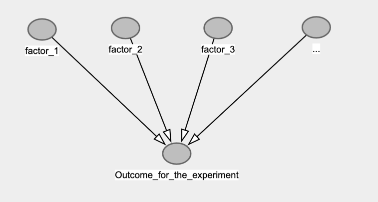

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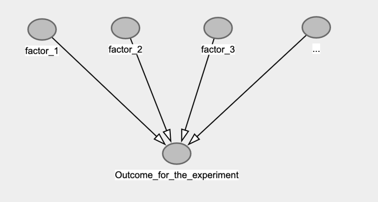

- A probability distribution is one way to summarize the effects from unmeasured factors: What is the chance (probability) of seeing more than 4 consecutive heads?
- If a game says you would gain $10 for less or equal than 4 consecutive heads and lose $30 for more than 4 consecutive heads, what amount of money would you gain in the long run?

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- What are the possible numbers implied by "seeing more than 4 consecutive heads"?
  - Let A records the possible number of consecutive heads that are greater than 4
  - $A = \{5, 6, 7, 8, 9, 10\}$
  - A is termed as **an event** of the sample space

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- $P(A) = p_5 + p_6 + p_7 + p_8 + p_9 + p_{10}$

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- The amount of money you would gain in the long run is $10 * P(X = 10) - 30 * P(X = 30)$. This is called the **expectation** of the random variable X, denoted by $E(X)$.

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- Example: Toss a Die
  - Valid sample space: {Even, Odd}-all outcomes and each outcome is unique
  - Invalid sample space:
    - {4 or greater}–Does not have all outcomes: 1, 2, and 3 not included
    - {4 or Greater, 5 or less}–Outcomes not unique: 4 and 5 are in both outcomes
- Failure to understand that outcomes must be unique has led to many incorrect analyses

More complicated sample space

- Consider an experiment of recording the detailed outcomes of flipping a coin three times. Sample space is $S = \{< TTT >, < HTT >, < THT >, < TTH >, < HHT >, < HTH >, < THH >, < HHH >\}$

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  - Sample space is a set $\{\}$ of tuples $<>$
    - Set$\{\}$: no repeats, order does not matter
    - Tuple $<>$: repeats allowed, order does matter unless we require order does not matter; $<>$ around outcomes can be omitted
- Consider an experiment of recording the battery lifespan of a randomly selected iphone. The sample space is probably $S = \{x|0 < x < 10\}$ in years; this represents an interval covering any number between 0 and 10, including integers and non-integers

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- Specify the sample space most appropriate for the problem

Sample space

If you cannot describe the sample space, you do not understand the problem.

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  - Events are NOT mutually exclusive–A and B share outcome 4
  - Outcomes ARE mutually exclusive–Outcomes sometimes called “elementary events”

Exercise

Example

Computer games in which the players take the roles of characters are very popular. They go back to earlier tabletop games such as Dungeons and Dragons. These games use many different types of dice. A four-sided die has faces with 1, 2, 3, and 4 spots.

(a) What is the sample space for the detailed outcomes of rolling a four-sided die twice (spots on first and second rolls)?

(b) List the outcomes in event A, for which the sum of rolling a four-sided die twice to be 4

(c) List the outcomes in event B, for which the sum of rolling a four-sided die twice to be 6

Answers to the exercise

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- It is fundamental to be able to operate sets. For example, we wish to express the event C for the maximum number of consecutive heads is less than 3 or greater than 6. There are three way to represent it
  - $C = \{1, 2, 7, 8, 9, 10\}$
  - Define $A = \{4, 5\}$. Then, $C = A^c$.
  - Define $A = \{1, 2\}$ and $B = \{7, 8, 9, 10\}$. Then $C = A \cup B$.

Review of sets

- **Union**: The union of A and B, written $A \cup B$ is the set of elements that belong to either A or B or both:

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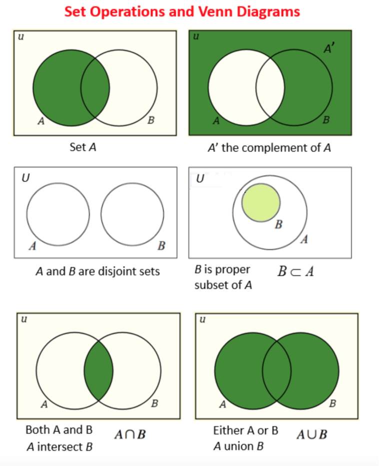
- **Complementation**: The complement of A, written A^c is the set of all elements that are not in A:

$$A^c = \{x : x \notin A\}$$

Review of sets using Venn diagram



Exercise

Given any sets B and A, use the Venn diagram show that $B = (B \cap A) \cup (B \cap A^c)$.

Review the following theorems and apply them to homework problems. For any three events, A, B, and C, defined on a sample space S,

- Commutativity: $A \cup B = B \cup A$; $A \cap B = B \cap A$.
- Associativity: $A \cup (B \cup C) = (A \cup B) \cup C$; $A \cap (B \cap C) = (A \cap B) \cap C$
- Distributive Laws: $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$; $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.
- DeMorgan's Laws: $(A \cup B)^c = A^c \cap B^c$; $(A \cap B)^c = A^c \cup B^c$.

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- **Continuous sample space** : A sample space is continuous if it contains an interval of real numbers (including integers and non-integers), for example, $S = \{x|2 < x < 10\}$

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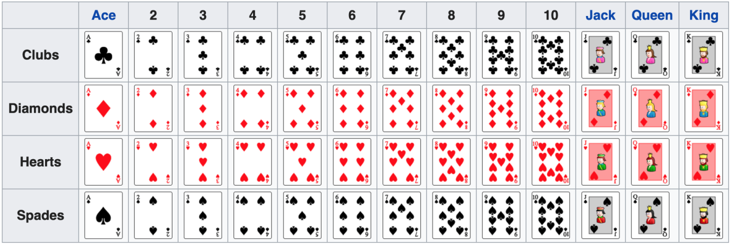
- If all outcomes in an event are equal likely, then knowing the probability of one of them and the total number of outcomes in the event, we can know the probability of the event.
  - If we flip an unfair coin twice (chance of head is 0.6), the probability of ‘HT’ is 0.6*0.4, and the probability of ‘TH’ is 0.4*0.6. Define event A as showing up exactly one head among two flips. Then $P(A) = 2 * 0.4 * 0.6 = 0.48$.

Example set of 52 playing cards; 13 of each suit clubs, diamonds, hearts, and spades



Example

Game 1- shuffle four cards: a diamond, a heart, a spade, and a club; then play the first two cards in order. To win the game, the first card needs to be a diamond or a heart and the second card needs to be a spade or a club.

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- Suppose all cards are well-shuffled. What is the chance of winning the game?

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Counting techniques

- If a job consists of k separate tasks, the *i*th of which can be done in n_i ways, $i = 1, \ldots, k$, then the entire job can be done in $n_1 \times n_2 \times \ldots n_k$ ways.

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- Permutation rule:
  - A permutation is an arranging of a subset of items from a collection into a sequence where the order of items does not matter
  - The number of permutations of size r elements selected from a set of n different elements is is

$$P_r^n = n \times (n-1) \times (n-2) \times \cdots \times (n-r+1) = \frac{n!}{(n-r)!}$$

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- Combination rule:
  - A combination is a subset of items from a collection where the order of items does not matter
  - The number of combinations of size r selected from a set of n elements, is denoted as $\binom{n}{r}$ or C_r^n and

$$C_r^n = \binom{n}{r} = \frac{n!}{r!(n-r)!}.$$

Permutation vs combination

Selecting a subset of three letters out of $\{A, B, C, D, E\}$. The number of possible combinations of size 3 is $\binom{5}{3} = C_3^5 = 10$. The number of permutations of size 3 is $P_3^5 = 60$.

	Combinations	Permutations						
$_5C_3$ of these	{A, B, C} →	ABC	BCA	CAB	CBA	BAC	ACB	$3! \cdot {}_5C_3$ of these
	{A, B, D} →	ABD	BDA	DAB	DBA	BAD	ADB	
	{A, B, E} →	ABE	BEA	EAB	EBA	BAE	AEB	
	{A, C, D} →	ACD	CDA	DAC	DCA	CAD	ADC	
	{A, C, E} →	ACE	CEA	EAC	ECA	CAE	AEC	
	{A, D, E} →	ADE	DEA	EAD	EDA	DAE	AED	
	{B, C, D} →	BCD	CDB	DBC	DCB	CBD	BDC	
	{B, C, E} →	BCE	CEB	EBC	ECB	CBE	BEC	
	{B, D, E} →	BDE	DEB	EBD	EDB	DBE	BED	
	{C, D, E} →	CDE	DEC	ECD	EDC	DCE	CED	

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- The number of possible way to win the game is $2 * 2 = 4$.
- Since all cards are shuffled well, the probabilities of playing any order of any two cards are equal. Hence the probability of winning the game is $4/12 = 1/3$.

Rework on Game 2

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Rework on Game 2

- The number of possible way to play the two cards is $\binom{5}{2} = C_2^4 = 6$.
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Rework on Game 2

- The number of possible way to play the two cards is $\binom{5}{2} = C_2^4 = 6$.
- The number of possible way to win the game is $\binom{3}{1} + \binom{3}{1} - 1 = 5$.
- Since all cards are shuffled well, the probabilities of playing any order of any two cards are equal. Hence the probability of winning the game is $5/6$.

More counting rules

- If event A and event B are mutually exclusive, then the number of outcomes in $A \cup B$ is the sum of the number of outcomes in A and the number of outcomes in B.

$$\#\text{ of } A \cup B = (\#\text{ of } A) + (\#\text{ of } B)$$

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- The number of outcomes in A^c is the the total number of outcomes in the space minus the number of outcome in event A:

$$\# \text{ of } A^c = (\# \text{ of } S) - (\# \text{ of } A)$$

Exercise

Example

A bin of 50 manufactured parts contains 3 defective parts and 47 non-defective parts. A sample of 6 parts is selected from the 50 parts without replacement. How many different samples are there of size 6 that contain less than 2 defective parts?

Answer to the exercise

Hint: Define B_0 as the event for selecting a sample of size 6 that has 0 defective part and B_1 as the event for selecting a sample of size 6 that has 1 defective part. Then the question becomes: how many outcomes in $B_0 \cup B_1$?

- To count B_1, notice that the job can be done in two steps
  - Step 1: choose 1 defective part from the pool of 3 defective parts–$\binom{3}{1}$ ways for this step,
  - Step 2: choose 5 defective parts from the pool of 47 non-defective parts–$\binom{47}{5}$ ways for this step,
  - Hence B_1 can be achieved in $\binom{3}{1} * \binom{47}{5}$ ways.
- Similarly B_0 can be achieved in $\binom{47}{6}$ ways.
- Finally, A can be achieved in $\binom{3}{1} * \binom{47}{5} + \binom{47}{6}$ ways.