STAT481/581: Introduction to Time Series Analysis

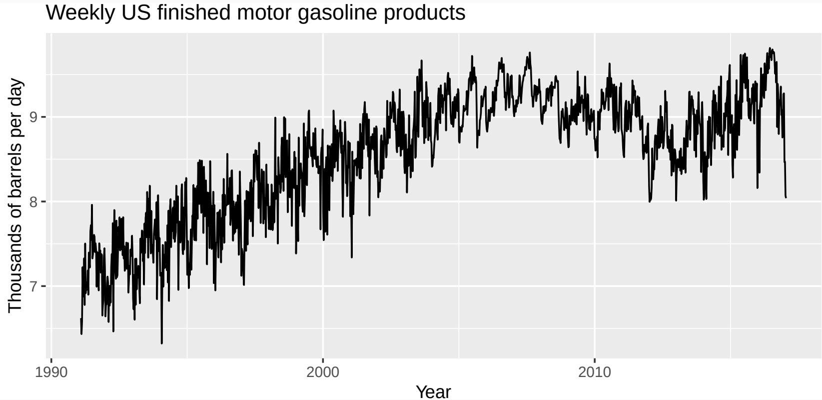
Advanced forecasting methods

OTexts.org/fpp3/

Outline

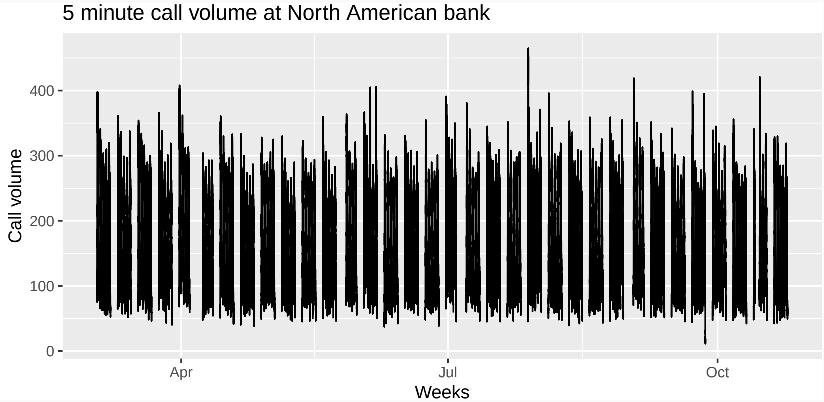
Outline

Examples

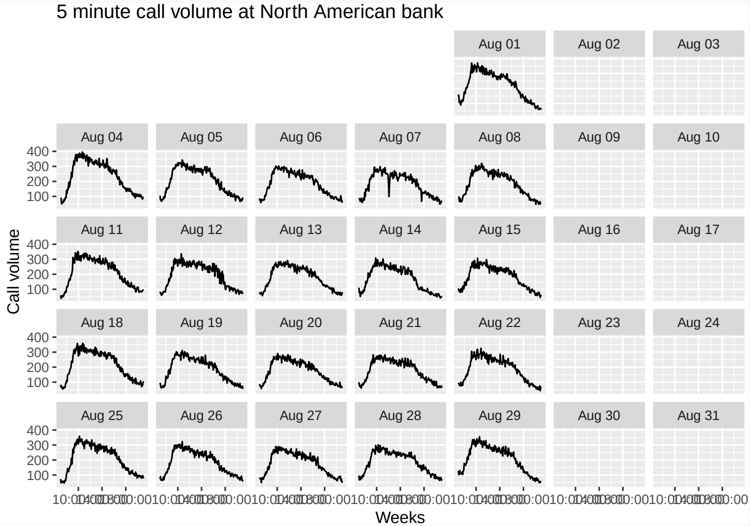


Weekly US finished motor gasoline products
Thousands of barrels per day
9
8
7
1990
2000
2010
Year

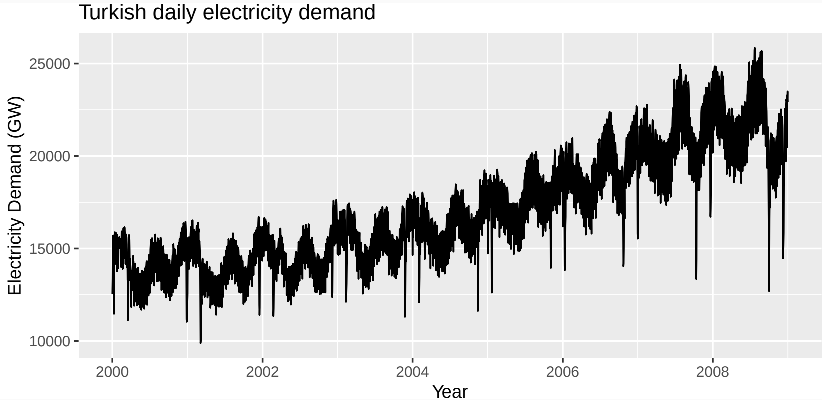
Examples


5 minute call volume at North American bank
Call volume
400
300
200
100
0
Apr
Jul
Oct
Weeks

Examples


5 minute call volume at North American bank
Aug 01
Aug 02
Aug 03
Aug 04
Aug 05
Aug 06
Aug 07
Aug 08
Aug 09
Aug 10
Aug 11
Aug 12
Aug 13
Aug 14
Aug 15
Aug 16
Aug 17
Aug 18
Aug 19
Aug 20
Aug 21
Aug 22
Aug 23
Aug 24
Aug 25
Aug 26
Aug 27
Aug 28
Aug 29
Aug 30
Aug 31
400
300
200
100
Call volume
Weeks

Examples


Turkish daily electricity demand
Electricity Demand (GW)
25000
20000
15000
10000
2000
2002
2004
2006
2008
Year

TBATS model

TBATS

Trigonometric terms for seasonality
Box-Cox transformations for heterogeneity
ARMA errors for short-term dynamics
Trend (possibly damped)
Seasonal (including multiple and non-integer periods)

TBATS model

$$y_t = \text{observation at time } t$$

$$y_t^{(\omega)} = \begin{cases} (y_t^\omega - 1)/\omega & \text{if } \omega \neq 0; \\ \log y_t & \text{if } \omega = 0. \end{cases}$$

$$y_t^{(\omega)} = \ell_{t-1} + \phi b_{t-1} + \sum_{i=1}^{M} s_{t-m_i}^{(i)} + d_t$$

$$\ell_t = \ell_{t-1} + \phi b_{t-1} + \alpha d_t$$

$$b_t = (1-\phi)b + \phi b_{t-1} + \beta d_t$$

$$d_t = \sum_{i=1}^{p} \phi_i d_{t-i} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} + \varepsilon_t$$

$$s_t^{(i)} = \sum_{j=1}^{k_i} s_{j,t}^{(i)} \qquad \begin{aligned} s_{j,t}^{(i)} &= s_{j,t-1}^{(i)} \cos \lambda_j^{(i)} + s_{j,t-1}^{*(i)} \sin \lambda_j^{(i)} + \gamma_1^{(i)} d_t \\ s_{j,t}^{(i)} &= -s_{j,t-1}^{(i)} \sin \lambda_j^{(i)} + s_{j,t-1}^{*(i)} \cos \lambda_j^{(i)} + \gamma_2^{(i)} d_t \end{aligned}$$

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M seasonal periods

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global and local trend

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ARMA error

$$s_t^{(i)} = \sum_{j=1}^{k_i} s_{j,t}^{(i)} \qquad \begin{aligned} s_{j,t}^{(i)} &= s_{j,t-1}^{(i)} \cos\lambda_j^{(i)} + s_{j,t-1}^{*(i)} \sin\lambda_j^{(i)} + \gamma_1^{(i)} d_t \\ s_{j,t}^{*(i)} &= -s_{j,t-1}^{(i)} \sin\lambda_j^{(i)} + s_{j,t-1}^{*(i)} \cos\lambda_j^{(i)} + \gamma_2^{(i)} d_t \end{aligned}$$

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M seasonal periods

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ARMA error

$$d_t = \sum_{i=1}^{p} \phi_i d_{t-i} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} + \varepsilon_t$$

Fourier-like seasonal terms

$$s_t^{(i)} = \sum_{j=1}^{k_i} s_{j,t}^{(i)}$$

$$s_{j,t}^{(i)} = s_{j,t-1}^{(i)} \text{ co}$$

$$s_{j,t}^{*(i)} = -s_{j,t-1}^{(i)} \sin \lambda_j^{(i)} + s_{j,t-1}^{*(i)} \cos \lambda_j^{(i)} + \gamma_2^{(i)} d_t$$

TBATS model

$y_t =$ observation at time t

$y_t^{(\omega)} = \Big\{ (y_t^\omega - 1)/\omega \quad \text{if } \omega \neq 0;$

$y_t^{(\omega)} =$ … $+ d_t$

$\ell_t =$ …

$b_t =$ …

$d_t =$ … $+ \varepsilon_t$

TBATS

Trigonometric

Box-Cox

ARMA

Trend

Seasonal

Box-Cox transformation

M seasonal periods

global and local trend

ARMA error

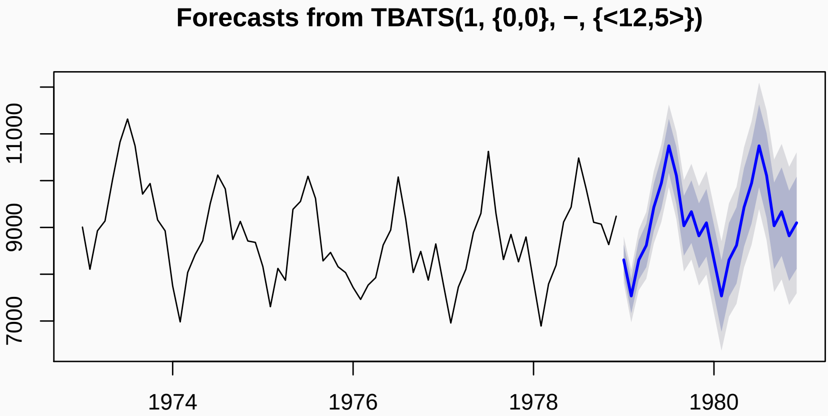
Fourier-like seasonal terms

$$s_t^{(i)} = \sum_{j=1}^{k_i} s_{j,t}^{(i)} \qquad s_{j,t}^{(i)} = s_{j,t-1}^{(i)} \cos \ldots$$

$$s_{j,t}^{(i)} = -s_{j,t-1}^{(i)} \sin \lambda_j^{(i)} + s_{j,t-1}^{*(i)} \cos \lambda_j^{(i)} + \gamma_2^{(i)} d_t$$

Complex seasonality

```r
fit <- tbats(USAccDeaths)
plot(forecast(fit))
```



TBATS model

TBATS

Trigonometric terms for seasonality

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ARMA errors for short-term dynamics

Trend (possibly damped)

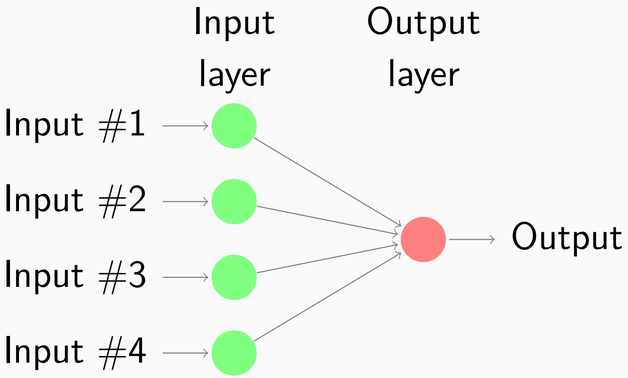
Seasonal (including multiple and non-integer periods)

- Handles non-integer seasonality, multiple seasonal periods.
- Entirely automated
- Prediction intervals often too wide
- Very slow on long series

Outline

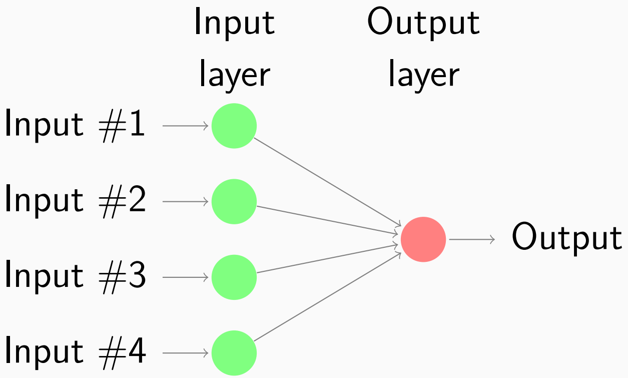
Neural network models

Simplest version: linear regression



Neural network models

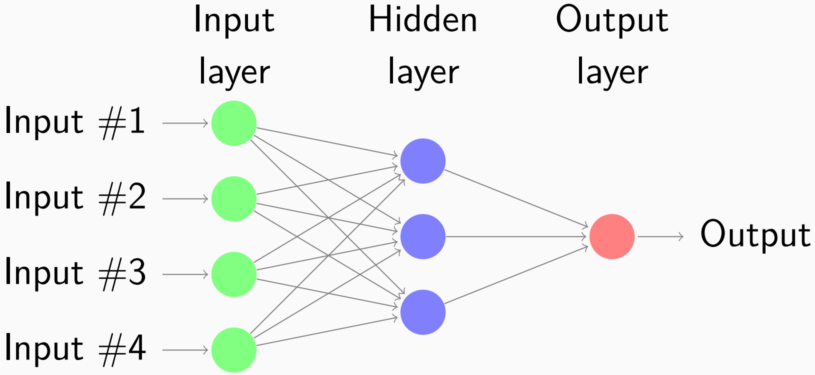
Simplest version: linear regression



- Coefficients attached to predictors are called “weights”.
- Forecasts are obtained by a linear combination of inputs.
- Weights selected using a “learning algorithm” that

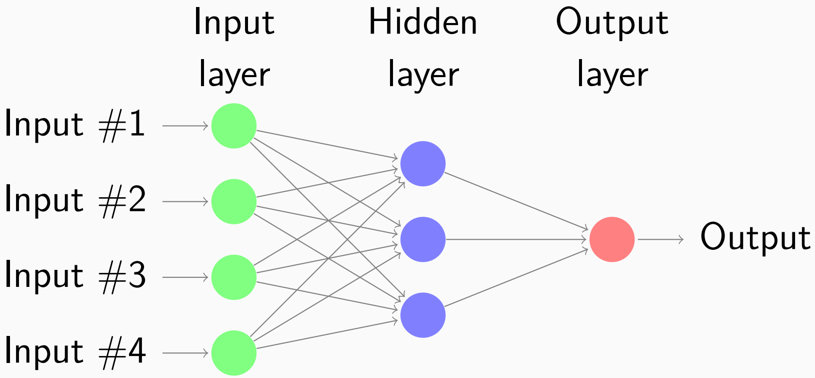
Neural network models

Nonlinear model with one hidden layer



Neural network models

Nonlinear model with one hidden layer



- A **multilayer feed-forward network** where each layer of nodes receives inputs from the previous

Neural network models

Inputs to hidden neuron j linearly combined:

$$z_j = b_j + \sum_{i=1}^{4} w_{i,j} x_i.$$

Modified using nonlinear function such as a sigmoid:

$$s(z) = \frac{1}{1 + e^{-z}},$$

This tends to reduce the effect of extreme input values, thus making the network somewhat robust to outliers.

Neural network models

- Weights take random values to begin with, which are then updated using the observed data.
- There is an element of randomness in the predictions. So the network is usually trained several times using different random starting points, and the results are averaged.
- Number of hidden layers, and the number of nodes in each hidden layer, must be specified in advance.

NNAR models

- Lagged values of the time series can be used as inputs to a neural network.
- NNAR(p, k): p lagged inputs and k nodes in the single hidden layer.
- NNAR($p, 0$) model is equivalent to an ARIMA($p, 0, 0$) model but without stationarity restrictions.
- Seasonal NNAR(p, P, k): inputs $(y_{t-1}, y_{t-2}, \dots, y_{t-p}, y_{t-m}, y_{t-2m}, y_{t-Pm})$ and k neurons in the hidden layer.
- NNAR$(p, P, 0)_m$ model is equivalent to an ARIMA$(p, 0, 0)(P,0,0)_m$ model but without stationarity restrictions.

NNAR models in R

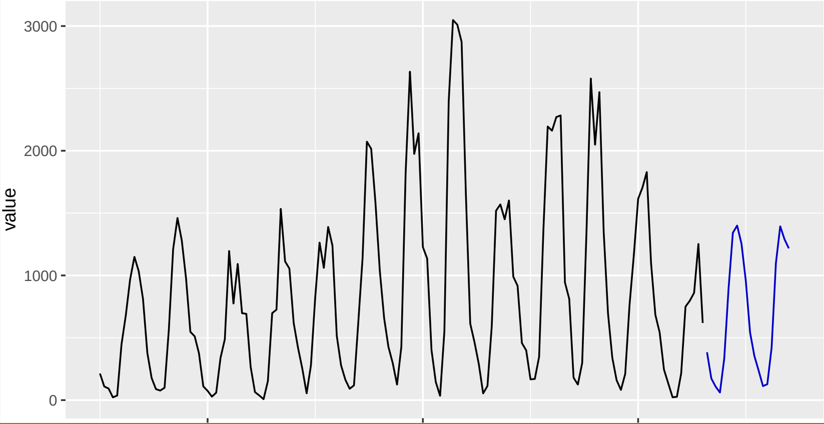
- The `nnetar()` function fits an NNAR$(p, P, k)_m$ model.
- If p and P are not specified, they are automatically selected.
- For non-seasonal time series, default $p =$ optimal number of lags (according to the AIC) for a linear AR(p) model.
- For seasonal time series, defaults are $P = 1$ and p is chosen from the optimal linear model fitted to the seasonally adjusted data.
- Default $k = (p + P + 1)/2$ (rounded to the nearest integer).

Sunspots

- Surface of the sun contains magnetic regions that appear as dark spots.
- These affect the propagation of radio waves and so telecommunication companies like to predict sunspot activity in order to plan for any future difficulties.
- Sunspots follow a cycle of length between 9 and 14 years.

NNAR(9,5) model for sunspots

```
sunspots <- as_tsibble(fpp2::sunspotarea)
fit <- sunspots %>% model(NNETAR(value))
fit %>% forecast(h=20, times = 1) %>%
  autoplot(sunspots, level = NULL)
```



Prediction intervals by simulation

```r
fit %>% forecast(h=20) %>%
  autoplot(sunspots)
```