MATH 316
Ordinary Differential Equations (ODEs)
MWF 10:00-10:50 & MWF 2:00-2:50

Instructor: Lane McConnell
Office Hours: M 11:00-1:00 and W 3:00-4:00 (or by appointment) in main lobby of SMLC
Office: SMLC 332
Email: lmcconne@math.unm.edu
URL: `http://math.unm.edu/~lmcconne/`
Texts: Brannan and Boyce (BB), Differential Equations, (required)
Polking and Arnold (PA), Ordinary Differential Equations using Matlab, 3rd ed. (not required)

Course Objectives:

To put it briefly, the point of this class is to take your existing knowledge of calculus and apply it towards the construction and solution of mathematical models in the form of differential equations (i.e. equations with derivatives in them). That's it. More precisely, the goal is that by the end of the class you will be able to:

- *Understand* all of the concepts relating to the order and linearity of ODEs, analytic and computational solution methods for ODEs, and the real-world applications of ODEs.
- *Apply* your understanding of the concepts, formulas, and problem solving procedures to thoroughly investigate relevant physical models.
- *Explain* the concepts of linear systems, ODE solution methods, and related ideas at a fundamental level, as well as how and why we use the solution techniques that we use.

Assessment:

My goal in teaching this class is to provide you with a deep understanding of differential equations. This means developing the material in such a way that the concepts (i.e. what is a differential equation?) are connected to the procedures (i.e. how do you compute the solution to a differential equation?). Math isn't just magical formulas; everything in math is derived from some set of relatively common sense principles. I want you to understand not only how to use the formulas to solve problems, but also to understand where they come from and where they are useful. There are several ways I will monitor your progress to gauge your understanding of the material.

Homework will be assigned in each class, but will only be collected on a weekly basis (Fridays?). They will be short and (hopefully) not overly burdensome; I will try to pick out a few problems that cover the topics from class in some depth. Homeworks will also occasionally involve concept based questions that will require you to explain, in your own written words, a particular concept or procedure related to the material. That's right: this is basically a short math essay!

Midterms will provide a chance to evaluate your true understanding of the material. Each exam will focus primarily on the preceeding material, and will be designed to test your ability to apply specific problem solving tools and techniques as well as your general understanding of the fundamental concepts.

The final exam will be both cumulative and comprehensive. It will contain both questions that require you to compute exact solutions and general concept questions. As part of this, there will likely be short essay questions on topics related to the material.

Grading:

Homework: 30%
Midterms (3 total): 45% (15% each)
Final Exam: 25%

How to Succeed:

The material presented in this class is inherently difficult. I expect some of the concepts to seem complicated and confusing when they are first presented. I also expect you, the student, to respond accordingly. Here are some ways to make things easier on yourself:

- *Ask questions* about concepts and topics that seem unclear. Speak up if something doesnt make sense. It is very likely that if you have a question, someone else does too. Questions not only help you and the other students get the clarification you need, but they also help me to refocus and improve my teaching.

- *Collaborate* with the other students. Working together is a very important skill to cultivate (and one that will certainly serve you well throughout the entirety of your college careers). Your peers can provide you with a different point of view, explain concepts in a new manner, or simply lend a sympathetic ear to commiserate with. I would particularly encourage you

to work with other students on homework, be it talking about and trying to understand the concepts together, or simply checking answers. However, make sure to write up your own solutions presented in your own words and reflecting your own thoughts. Copying is NOT permitted!

- *Seek help* if you're feeling lost or behind. Attendance at office hours is highly recommended and shows me that you are truly invested in learning the material and improving on weaknesses. If nothing else, office hours allow me, your instructor, to meet you as an individual, rather than as just a face in the crowd.

Attendance Policy

Regular and punctual attendance is required. UNM Pathfinder policies apply, which in part means instructor drops based on non-attendance are possible. This policy applies regardless of the grading option you have chosen.

Accommodation Statement

Accessibility Services (Mesa Vista Hall 2021, 277-3506) provides academic support to students who have disabilities. If you think you need alternative accessible formats for undertaking and completing coursework, you should contact this service right away to assure your needs are met in a timely manner. If you need local assistance in contacting Accessibility Services, see the Bachelor and Graduate Programs office.

Academic Integrity

The University of New Mexico believes that academic honesty is a foundation principle for personal and academic development. All University policies regarding academic honesty apply to this course. Academic dishonesty includes, but is not limited to, cheating or copying, plagiarism (claiming credit for the words or works of another from any type of source such as print, Internet or electronic database, or failing to cite the source), fabricating information or citations, facilitating acts of academic dishonesty by others, having unauthorized possession of examinations, submitting work of another person or work previously used without informing the instructor, or tampering with the academic work of other students. The University's full statement on academic honesty and the consequences for failure to comply is available in the college catalog and in the Pathfinder.

Cell Phones and Technology

As a matter of courtesy, please turn off cell phones, pagers, and other communication and entertainment devices prior to the beginning of class. Notify me in advance if you are monitoring an emergency, for which cell phone ringers should be switched to vibrate.

Library and Tutorial Services

UNM-Main campus provides many library services and some tutorial services for distance students. For library services, go to http://www.unm.edu/libraries/ to link to a specific library or to contact a librarian. For tutorial services, go to http://caps.unm.edu/online to explore UNMs online services.

Course Outline

- First Order Equations $\frac{dy}{dx} = f(x, y)$
  Here the emphasis is on geometry, solution techniques and numerical approximations.
  - Direction fields, solutions and Euler's method
  - Linear $y' + p(x)y = g(x)$; integrating factor and variation of parameter
  - Separable equations $M(x) + N(y)\frac{dy}{dx} = 0$
  - Improved Euler and variable step methods
  - Phase line and stability for the autonomous case

  See Sections 1.1-1.3, 2.1-2, 2.5, 2.7-8 in BB and Chapters 3 and 5 in PA
  Optional material from other sections in Chapter 2 if time.

- Systems of First Order DEs 2x2 case
  $x' = ax + by$
  $y' = cx + dy$
  - Matrix formulation and elementary matrix manipulations
  - Eigen problem and general solution
  - Demonstration that the general solution (i.e., the set of all solutions) is a linear combination of two linearly independent solutions
  - Phase Plane
  - Fundamental Solution Matrix (briefly)
  - 4th order Runge Kutta

  Most of Chapter 3 (BB)

- Homogeneous Second Order Equations $ay'' + by' + cy = 0$
  - Constant coefficient homogeneous
  - Again, demonstration that the general solution in the homogeneous case is a linear combination of two linearly independent solutions

  Sections 4.1-4.4 (BB)

- Nonhomogeneous Second Order Equations $ay'' + by' + cy = g(t)$

- General solution = G.S. of homogeneous + particular solution
- Method of Undertermined Coefficients
- Variation of Parameters - Main point: it always works, but more complicated than UC
- Harmonic and forced harmonic motion

Sections 4.5-4.8 (BB)

- Laplace Transforms
  Chapter 5 (BB)

- Nonlinear Equations and Stability
  - Autonomous systems in the plane
    $x' = f(x, y); y' = g(x, y)$
    * Equilibrium solutions and stability
    * Linearization about equlibrium solutions
    * Phase plane portraits with emphasis on $x'' + g(x) = 0$ and the energy method