MATH 313, Complex Variables , Spring 2020
Prof. J. Lorenz, Instructor
Homework 1, assigned Jan 24, due Feb. 7

1. Write the number

$$z = \frac{100}{(1-i)(2-i)(3-i)}$$

in the form $z = x + iy$.

2. Prove the rule $\overline{z_1 z_2} = \bar{z}_1 \, \bar{z}_2$.

3. Prove the rule $|z_1||z_2| = |z_1 z_2|$.

4. Use the triangle inequality $|z+w| \le |z| + |w|$ to prove the estimate

$$\Big| |z_1| - |z_2| \Big| \le |z_1 - z_2| \ .$$

5. Recall that $\cos(\pi/3) = \frac{1}{2}$. You may use this to write the number $z = 1 + i\sqrt{3}$ in polar form, $z = re^{i\phi}$.

Also, write the number $w = -10 + i\sqrt{300}$ in polar form.

6. Write the number

$$z = (1 + i\sqrt{3})^7$$

in Cartesian form, $z = x + iy$.

7. Prove the estimate

$$|\mathrm{Re}\, z| + |\mathrm{Im}\, z| \le \sqrt{2}\, |z| \quad \text{for all} \quad z \in= \mathbb{C} \ .$$

8. Find the Cartesian representations, $z = x + iy$, of the following numbers: (a) $e^{2-3\pi i}$, (b) $e^{(2+\pi i)/4}$, (c) Log e, (d) Log i, (e) Log $(1 + \sqrt{3}\, i)$, (f) Log $(-ei)$, (g) Log $(1 - i)$.

9. Find the principle value of $z_1 = i^i$ and of $z_2 = (-1 - \sqrt{3}\, i)^{12}$.

10. a) Find the solutions of the equation

$$z^2 + z + 1 = 0$$

and write them in polar form. Sketch them in the complex plane. b) Do the same for the equation $z^4 + 1 = 0$.

11. Write the following numbers in polar form, $z = re^{i\phi}$.
a) $z = \frac{-2}{1+i\sqrt{3}}$; b) $z = \frac{i}{-2-2i}$; c) $z = (\sqrt{3} - i)^6$.

12. Sketch the line given by

$$z = 1 + iy, \quad -\infty < y < \infty \ ,$$

and the image of this line under the map $f(z) = z^2$.

13. What is the distance of $z = 2 - i$ from its negative, $-z$?

14. Sketch the set of all complex numbers z with $|z - 4| = |z - 3i|$.

15a) Sketch the line of all z with $|z| = 1 + \operatorname{Im} z$.
b) Sketch the line of all z with $|z| = 1 + \operatorname{Re} z$.

16. Let $z, w \in \mathbb{C}$ and assume that $|z| = 1$. Prove that $|z - w| = |1 - z\bar{w}|$.
Hint: First assume that $z = 1$.