MATH 313, Complex Variables , Spring 2020
Prof. J. Lorenz, Instructor
Homework 4, assigned March 13, due April 3

1. Determine the Taylor series of $f(z) = e^z$ centered at $z_0 = 1$. For which values of z does the series converge?

2. Let $f(z) =$ Log $(1 + z)$ for $|z| < 1$. Determine the Taylor series of $f(z)$ about $z_0 = 0$. Determine the radius of convergence of the Taylor series.

3. Recall that

$$\tanh z = \frac{e^z - e^{-z}}{e^z + e^z} \ .$$

a) At which points does the function $\tanh z$ have singularities? What kind of singularities? (removable or pole or essential?)

b) Write the functions $e^z - e^{-z}$ and $e^z + e^{-z}$ as powers series, centered at $z_0 = 0$. For which $z \in \mathbb{C}$ do the series converge?

c) Determine the first two non–zero terms of the Taylor series of $\tanh z$, centered at $z_0 = 0$.

d) What is the radius of convergence of the Taylor series of $\tanh z$, centered at $z_0 = 0$?

4. Recall from calculus that

$$\frac{d}{dx} \arctan x = \frac{1}{1+x^2} \quad \text{for} \quad x \in \mathbb{R} \ .$$

Write the function $1/(1+x^2)$ as a power series, centered at $x_0 = 0$, for x in some interval. Use this to obtain the Taylor series of $\arctan z$, centered at $z_0 = 0$. Determine the radius of convergence of this Taylor series.

5. Write the function

$$f(z) = \frac{1}{4z - z^2}$$

as a Laurent series for $0 < |z| < 4$.

6. Write the function

$$f(z) = \frac{1}{z^2(1-z)}$$

as a Laurent series for
a) $0 < |z| < 1$;
b) $1 < |z|$.