MATH 412, Nonlinear Dynamics and Chaos, Spring 2019
Prof. J. Lorenz, Instructor
Homework 1, assigned Jan. 25, 2019, due Feb. 8, 2019

1a) Determine the fixed points of the equation

$$x' = e^{-x} \sin x \ .$$

b) Which of the fixed points are stable? Which are unstable?
c) Sketch the solution $x(t)$ with initial condition

$$x(0) = 1$$

for $0 \le t < \infty$.
d) Sketch the solution $x(t)$ with initial condition

$$x(0) = -1$$

for $0 \le t < \infty$.

2a) Use calculus to determine the solution of the initial value problem

$$x' = 1 + x^2, \quad x(0) = 1 \ .$$

b) At which time $t^* > 0$ does the solution blow up?
c) Sketch the solution for $0 \le t < t^*$.

3) Consider the equation

$$x' = r + 12x - x^3$$

where $-\infty < r < \infty$ is a parameter.
a) Sketch the fixed points as a function of r for $-\infty < r < \infty$. Which fixed points are stable? Which are unstable?
b) What kind of bifurcations occur? At which values of r?

4) Consider the equation

$$x' = rx - \ln(1+x)$$

where r is a parameter. (Here it is assumed that $x(t) > -1$.)
a) How many fixed points does the equation have for $0 < r < 1$? What is their stability?
b) How many fixed points does the equation have for $r > 1$? What is their stability?
c) Sketch the fixed points as a function of $0 < r < \infty$.
d) What kind of bifurcation occurs at $r = 1$?

5) Consider the equation

$$x' = x + \frac{rx}{1+x^2}$$

where r is a parameter.

a) Determine all fixed points as a function of the parameter r where $-\infty < r < \infty$. Determine their stability.

b) Sketch the bifurcations diagram. What kind of bifurcation occurs?