

Homework 3, Math. 562

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Assigned: Feb. 24, 2023. Due: March 10, 2023

1) Let S denote the open half-strip

$$S = \{z = x + iy : -\frac{\pi}{2} < x < \frac{\pi}{2}, y > 0\} .$$

- a) Determine the image of S under the map $z \rightarrow \sin z$.
- b) Determine the image of the boundary ∂S of S under the map $z \rightarrow \sin z$.
- c) Is the function $f(z) = \sin z$ one-to-one on $\bar{S} = S \cup \partial S$?

2) Let

$$w(q) = \int_0^q (1 + \zeta)^{-1/2} (1 - \zeta)^{-1/2} d\zeta$$

for $q \in \mathbb{R}$. How do you have to define the roots so that the function $q \rightarrow w(q)$ for $q \in \mathbb{R}$ inverts the function $z \rightarrow \sin z$ on ∂S ?

3) Let

$$f(z) = \int_1^z (\zeta + 1)^{-2/3} (\zeta - 1)^{-2/3} d\zeta .$$

Choose the roots as in class. Determine the image of the extended real line $\mathbb{R} \cup \{\pm\infty\}$ under the map $z \rightarrow f(z)$.

4) Fix an angle α with $0 < \alpha < \pi$ and consider the function

$$f(z) = \int_0^z \zeta^{-(\pi-\alpha)/\pi} d\zeta .$$

Choose the root as in class.

- a) Describe the image of the real line under the map $z \rightarrow f(z)$.
- b) In this case it is not too difficult to describe the image of the open upper half-plane \mathbb{H} under f . You can parametrize the straight line from 0 to z by

$$\zeta(t) = zt, \quad 0 \leq t \leq 1 ,$$

for example. Describe the image of \mathbb{H} under the map f .