```
In[123]:= (* This produces something homeomorphic to a three-
           sphere.  Just the Gamma matrices with one rescaled *)

In[124]:= n = 4;

In[125]:= q = 2;

In[126]:= r = 1;

In[127]:= s = 1;

In[128]:= t = 1;

In[129]:= sigmax = {{0, 1}, {1, 0}};

In[130]:= sigmay = {{0, -ⅈ}, {ⅈ, 0}};

In[131]:= sigmaz = {{1, 0}, {0, -1}};

In[132]:= I2 = IdentityMatrix[2];

In[133]:= AA = q * KroneckerProduct[-sigmay, sigmax];

In[134]:= MatrixForm[AA]
```

Out[134]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 2 i \\ 0 & 0 & 2 i & 0 \\ 0 & -2 i & 0 & 0 \\ -2 i & 0 & 0 & 0 \end{pmatrix}$$

```
In[135]:= BB = r * KroneckerProduct[-sigmay, sigmay];

In[136]:= MatrixForm[BB]
```

Out[136]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

```
In[137]:= CC = s * KroneckerProduct[-sigmay, sigmaz];

In[138]:= MatrixForm[CC]
```

Out[138]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & i & 0 \\ 0 & 0 & 0 & -i \\ -i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \end{pmatrix}$$

```
In[139]:= DD = t * KroneckerProduct[sigmax, I2];

In[140]:= MatrixForm[DD]
```

Out[140]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

In[141]:=
```
loclzrHalf = KroneckerProduct[ⅈ * sigmax, AA - w * IdentityMatrix[4]] +
   KroneckerProduct[ⅈ * sigmay, BB - x * IdentityMatrix[4]] +
   KroneckerProduct[ⅈ * sigmaz, CC - y * IdentityMatrix[4]] +
   KroneckerProduct[I2, DD - z * IdentityMatrix[4]];

charpoly = Factor[Det[loclzrHalf]]
```

Out[142]= $\left(w^4 + 2\, w^2 \left(-3 + x^2 + y^2 + z^2\right) + \left(3 + x^2 + y^2 + z^2\right)^2\right) \left(w^4 + 2\, w^2 \left(1 + x^2 + y^2 + z^2\right) + \left(-1 + x^2 + y^2 + z^2\right) \left(15 + x^2 + y^2 + z^2\right)\right)$

In[143]:=
```
(*Notice this involves only  w  and R = Sqrt[x^2 + w^2 + z^2]  is the result of a
  curve rotated into two more dimensions *)
```

In[144]:=
```
curvePoly = Factor[ReplaceAll[charpoly, {y^2 → 0, z^2 → 0, x^2 → R^2}]]
```

Out[144]= $\left(9 + 6\, R^2 + R^4 - 6\, w^2 + 2\, R^2\, w^2 + w^4\right) \left(-15 + 14\, R^2 + R^4 + 2\, w^2 + 2\, R^2\, w^2 + w^4\right)$

In[145]:=
```
factors = FactorList[curvePoly]
```

Out[145]= $\{\{1, 1\}, \{9 + 6\, R^2 + R^4 - 6\, w^2 + 2\, R^2\, w^2 + w^4, 1\}, \{-15 + 14\, R^2 + R^4 + 2\, w^2 + 2\, R^2\, w^2 + w^4, 1\}\}$

In[146]:=
```
p = factors[[2, 1]]
```

Out[146]= $9 + 6\, R^2 + R^4 - 6\, w^2 + 2\, R^2\, w^2 + w^4$

In[147]:=
```
p2 = ReplaceAll[p, {R → Sqrt[R], w → Sqrt[w]}]
```

Out[147]= $9 + 6\, R + R^2 - 6\, w + 2\, R\, w + w^2$

In[148]:=
```
q = factors[[3, 1]]
```

Out[148]= $-15 + 14\, R^2 + R^4 + 2\, w^2 + 2\, R^2\, w^2 + w^4$

In[149]:=
```
Solve[p2 == 0, R]
```

Out[149]= $\{\{R \to -3 - 2\sqrt{3}\,\sqrt{w} - w\}, \{R \to -3 + 2\sqrt{3}\,\sqrt{w} - w\}\}$

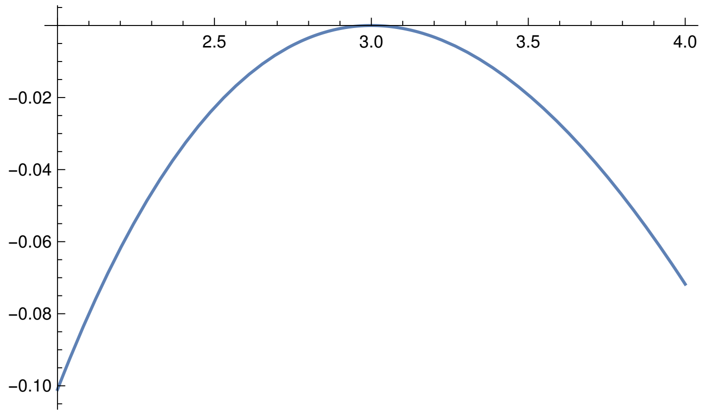
In[150]:= $-3 + 2\sqrt{3}\,\sqrt{w} - w$

Out[150]= $-3 + 2\sqrt{3}\,\sqrt{w} - w$

In[151]:= $\partial_w\left(-3 + 2\sqrt{3}\,\sqrt{w} - w\right)$

Out[151]= $-1 + \frac{\sqrt{3}}{\sqrt{w}}$

In[152]:=
```
Plot[-3 + 2 √3 √w - w, {w, 2, 4}]
```

Out[152]=


In[153]:=
```
(* We have one non-negative solution to  p2==0  so there is just one solution to
    p == 0, namely w = √3  and  R = 0.  Let's check this is a solutoin.
*)
```

In[154]:=
```
ReplaceAll[p, {w → Sqrt[3], R → 0}]
```

Out[154]= 0

In[155]:=
```
ReplaceAll[q, {w → Sqrt[3], R → 0}]
```

Out[155]= 0

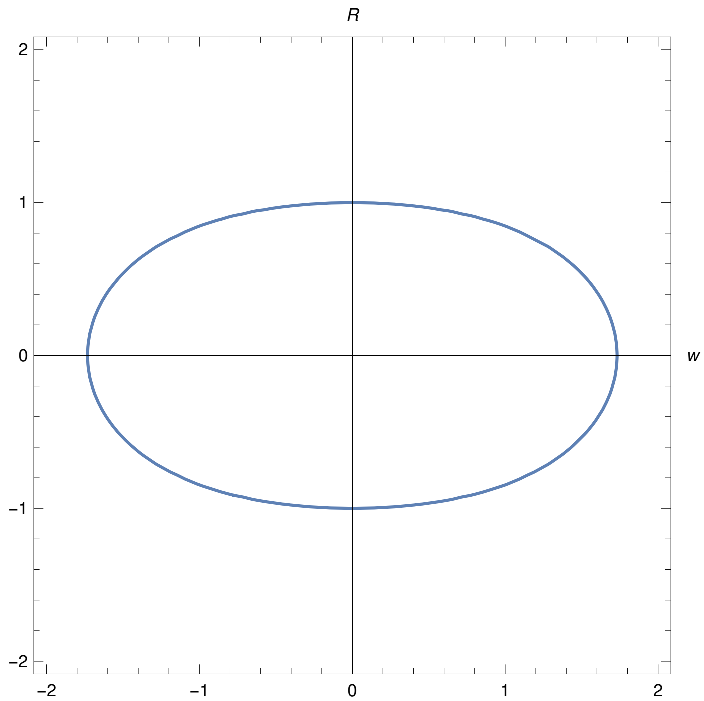
In[156]:=
```
ReplaceAll[curvePoly, {w → Sqrt[3], R → 0}]
```

Out[156]= 0

In[157]:=
```
(* Since this point is on the other curve, it adds nothing. *)
```

In[158]:=
```
fullPlot =
 ContourPlot[q == 0, {w, -2, 2}, {R, -2, 2}, Axes → Automatic, AxesLabel → {w, R}]
```

Out[158]=



In[159]:=
```
Export["AlmostThreeSphere.eps", fullPlot , ImageSize → 3.2 * 72];
```