HOMEWORK #4

Problem 1. Let f and g be the functions from $\{0, 1, 2, 3\}$ to $\{0, 1, 2, 3\}$ defined by
$$f(0) = 1, \quad f(1) = 0, \quad f(2) = 3, \text{ and } f(3) = 2$$
and
$$g(0) = 2, \quad g(1) = 3, \quad g(2) = 0, \text{ and } g(3) = 1.$$
Find the following functions, describing them in this way ($f(0) = *, \ldots$).

(a)
$$(f \circ g)^{-1}$$
(b)
$$\left(g^{-1} \circ f^{-1}\right)$$
(c)
$$\left(f^{-1} \circ g^{-1}\right)$$

Problem 2. Let f and g be the functions from $\mathbb{R} \setminus \{0\}$ to $\mathbb{R} \setminus \{0\}$ defined by
$$f(x) = 2x$$
and
$$g(x) = -x^{-1}.$$
Find formulas for:

(a)
$$(f \circ g)^{-1}(x)$$
(b)
$$\left(g^{-1} \circ f^{-1}\right)(x)$$
(c)
$$\left(f^{-1} \circ g^{-1}\right)(x)$$

Problem 3. Suppose f and h are the following functions from $\{0, 1, 2, 3\}$ to $\{0, 1, 2, 3\}$:

$$f = \big\{(0, 1), (1, 2), (2, 3), (3, 0)\big\}$$

and

$$h = \big\{(0, 1), (1, 1), (2, 3), (3, 2)\big\}$$

(a) Find $h \circ f$, giving your answer in the form of a set of ordered pairs.
(b) Find all possible functions that g can possibly be if we require that

$$g : \{0, 1, 2, 3\} \to \{0, 1, 2, 3\}$$

and

$$h \circ g = h \circ f.$$

Problem 4. Find two different ordered triples of natural numbers

$$(k, m, n) \neq (r, s, t)$$

so that

$$24^k 54^m 36^n = 24^r 54^s 36^t.$$

Problem 5. Find the greatest common divisors of each pair:

(a) $1000001, 3000013$
(b) $3^{23} \cdot 5^{34}, 3^{25} \cdot 5^{30}$
(c) $7423, 6281.$