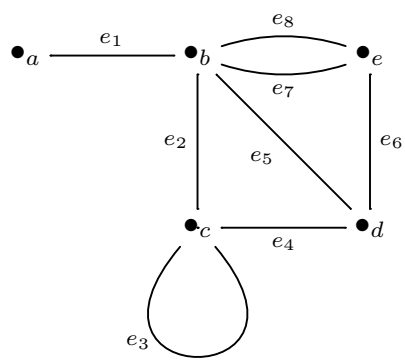
DEFINTION OF A GRAPH

0.1. **The incidence Function.** The definition of a graph needs to have three things: a set of vertices, a set of edges, and something to describe where the edges go. In the book, they give this example,



which they say has vertex set

$$V = \{a, b, c, d, e\}$$

and edge set

$$E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}$$

and where the ends of the edges are given by

$$\begin{array}{ll} e_1 \leftrightarrow (a, b) & e_5 \leftrightarrow (b, d) \\ e_2 \leftrightarrow (b, c) & e_6 \leftrightarrow (d, e) \\ e_3 \leftrightarrow (c, c) & e_7 \leftrightarrow (b, e) \\ e_4 \leftrightarrow (c, d) & e_8 \leftrightarrow (b, e) \end{array}$$

and where the pairs (v, w) are all *unordered* pairs. I have two small complaints.

You can't figure which edge a pair of vertices came from, so the bidirectional arrows are misleading. It is more accurate to say the graph contains this information:

$$\begin{array}{ll} e_1 \mapsto (a, b) & e_5 \mapsto (b, d) \\ e_2 \mapsto (b, c) & e_6 \mapsto (d, e) \\ e_3 \mapsto (c, c) & e_7 \mapsto (b, e) \\ e_4 \mapsto (c, d) & e_8 \mapsto (b, e) \end{array}$$

Secondly, the notation (x, y) for an *ordered* pair is so standard, so it is tricky to use the same notation for an unordered pair. We don't need any new notation anyway; we are talking about sets that have one or two elements. So, even better is

$$\begin{array}{ll} e_1 \mapsto \{a, b\} & e_5 \mapsto \{b, d\} \\ e_2 \mapsto \{b, c\} & e_6 \mapsto \{d, e\} \\ e_3 \mapsto \{c, c\} & e_7 \mapsto \{b, e\} \\ e_4 \mapsto \{c, d\} & e_8 \mapsto \{b, e\} \end{array}$$

or

$$\begin{array}{cc} e_1 \mapsto \{a,b\} & e_5 \mapsto \{b,d\} \\ e_2 \mapsto \{b,c\} & e_6 \mapsto \{d,e\} \\ e_3 \mapsto \{c\} & e_7 \mapsto \{b,e\} \\ e_4 \mapsto \{c,d\} & e_8 \mapsto \{b,e\} \end{array}$$

This table is describing a function, ι from E to V, called the incidence function.

0.2. **Equality.** Two graphs are be *equal* if the have:

(a) the same vertices
(b) the same edges
(c) the same incidence information/function

Example 0.1. Draw all the possible graphs that have vertex set

$$\{v, w\}$$

and edge set

$$\{e, f\}.$$

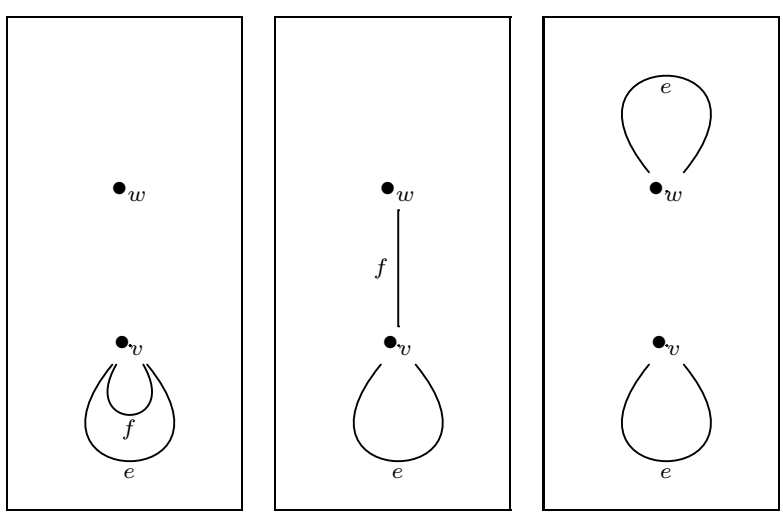
Solution: Each edge needs to be given one or two vertices to which to be incident. The choices are

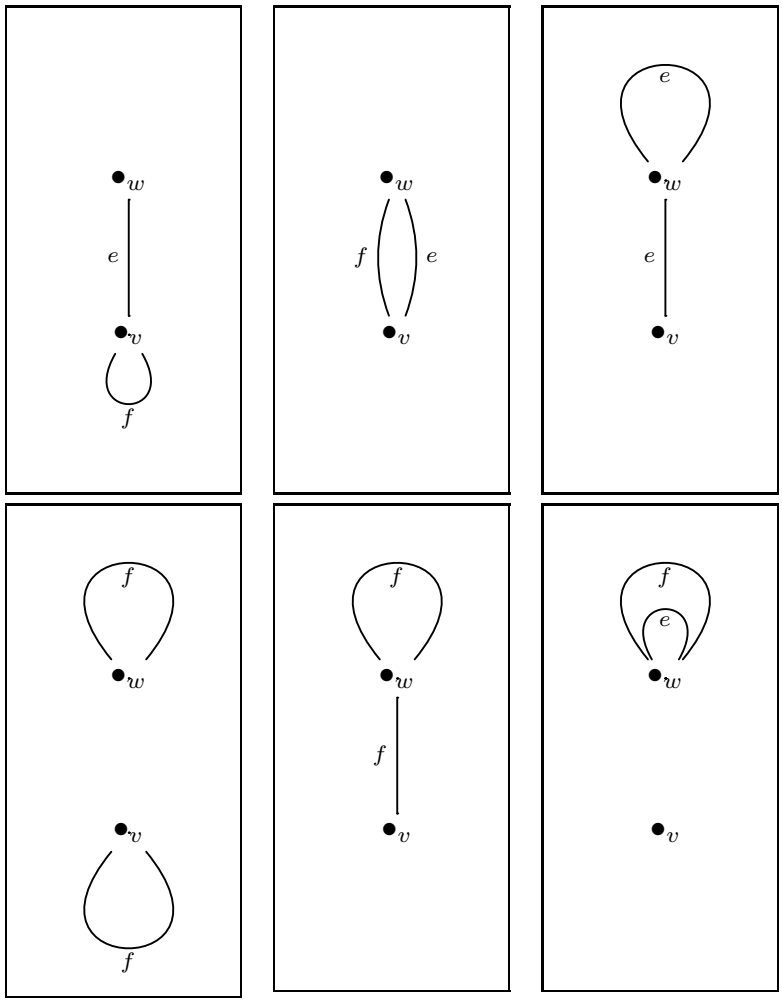
$$\text{just } v,$$

$$\text{just } w,$$

$$v \text{ and } w.$$

So there are a total of 9 possibilities:

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