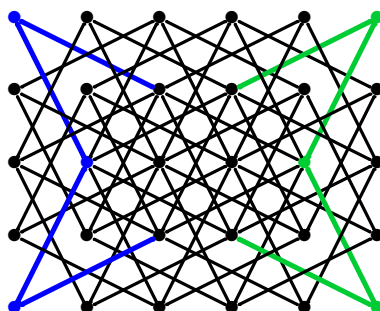
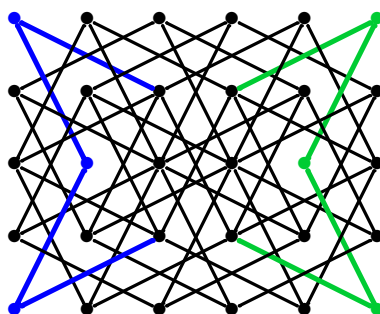


KNIGHTS TOUR

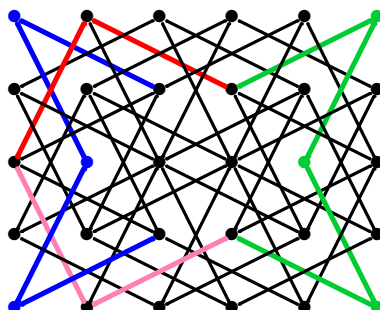
On the 5-by-6 chess board, we are forced to use the edges at the corners, as the degree of these vertices is two.



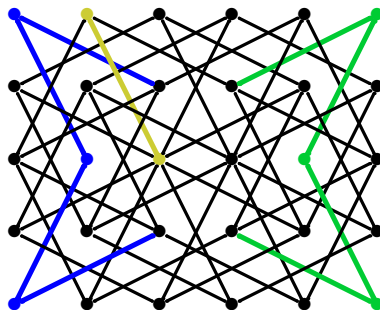
We cannot select any more edges at the the vertices where two are already selected, so we may remove these edges. This we shall call “pruning.”



Consider the two red and two pink edges:

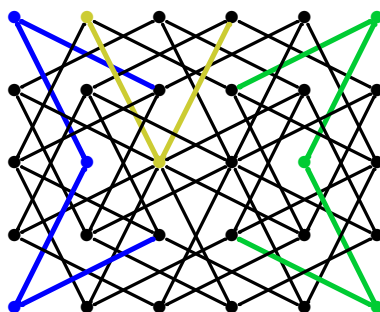


They cannot all be included. Therefore, up to symmetry, we can assume one of the red edges is omitted. It follows that the yellow edge must be included.

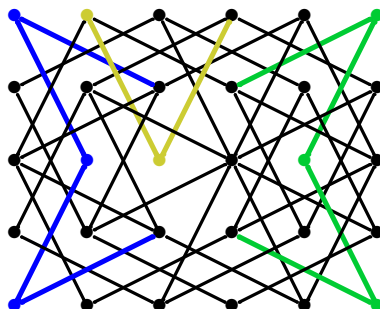


We consider the seven other edges at the degree-8 end of this edge.

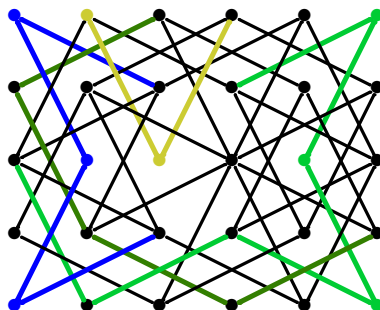
0.0.1. *Case I:*



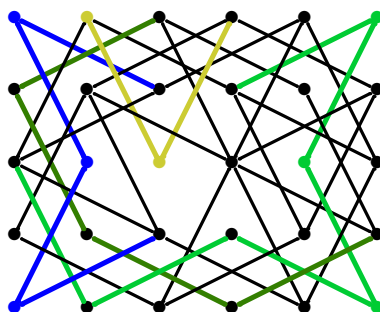
Prune:



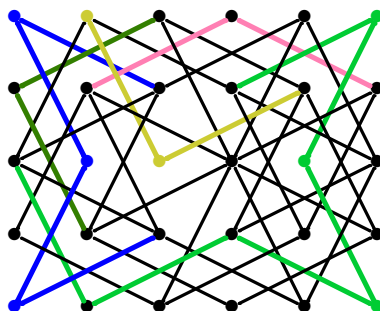
Select:



Prune:

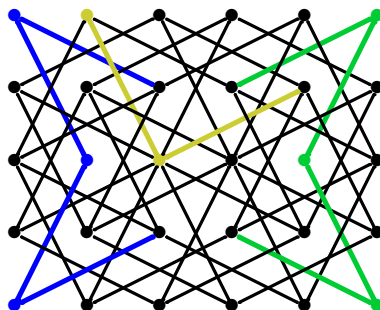


Select:

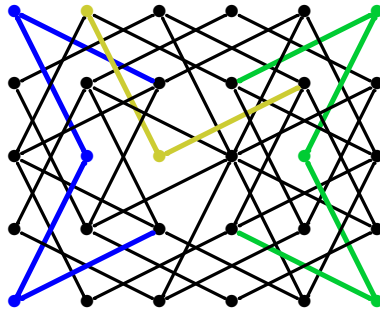


That's a 6-cycle, and so a dead end.

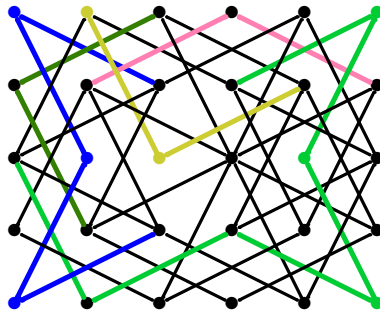
0.0.2. *Case II:*



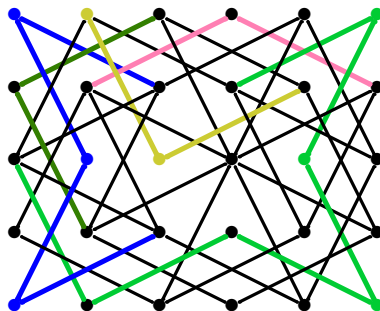
Prune:



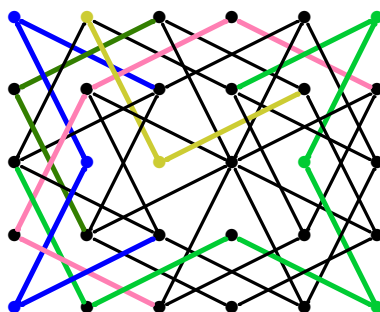
Select:



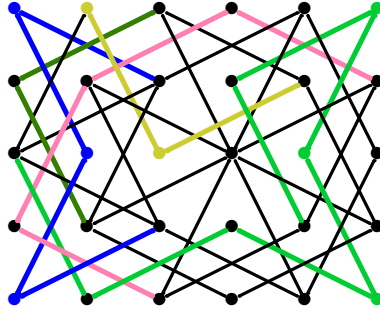
Prune:



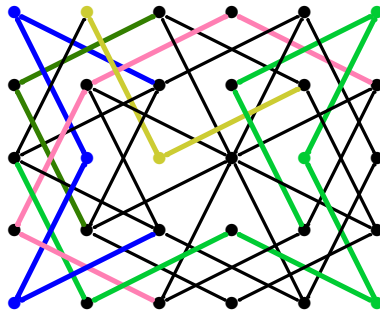
Select:



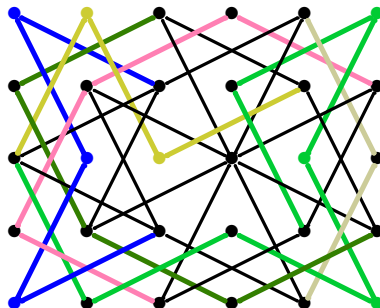
To avoid this pink 6-cycle, we must instead use red edge:



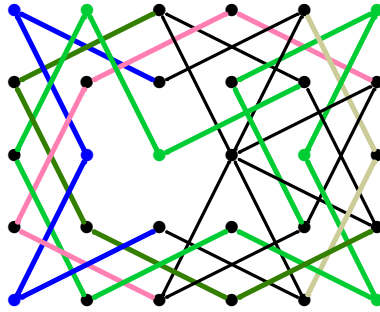
Recolor and prune:



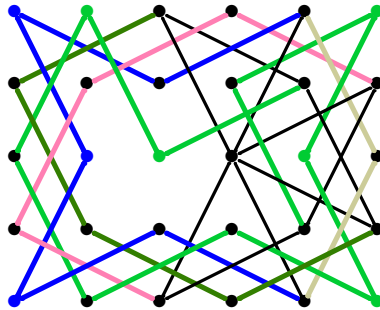
Select the edges where the degree is 2 :



Color and prune:

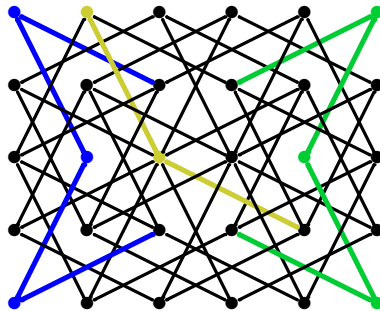


Select:

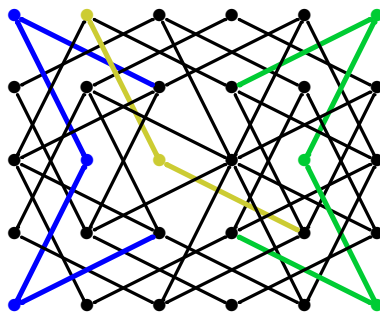


That made a 6-cycle, so we are at another dead end.

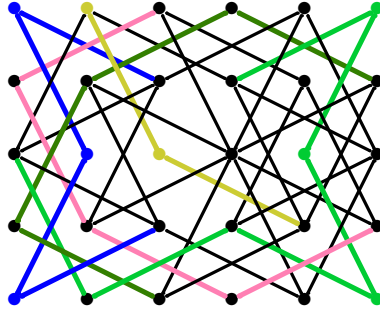
0.0.3. *Case III*:



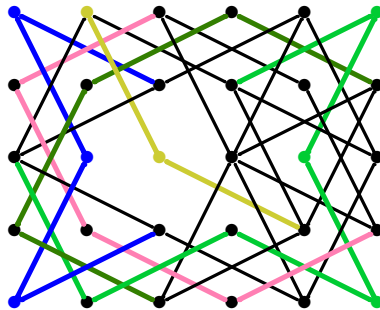
Prune:



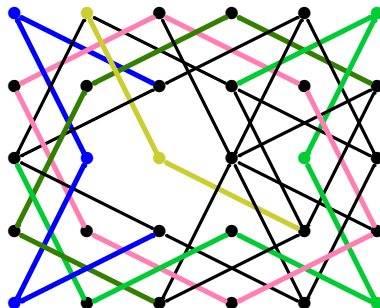
Select where the degree is 2 :



Prune:

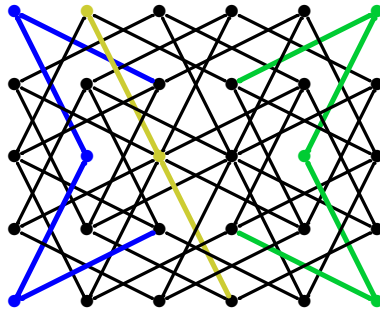


This forces us to chose a 6-cycle:

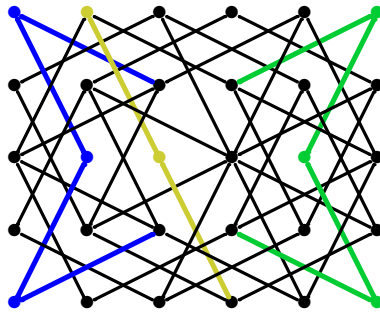


Another dead end.

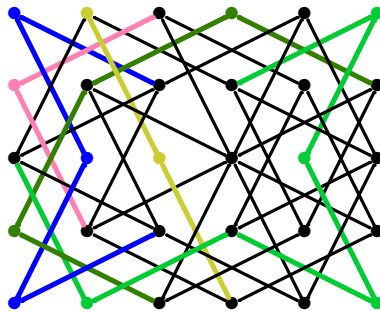
0.0.4. *Case IV*..



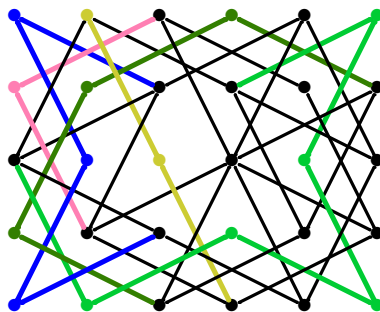
Prune:



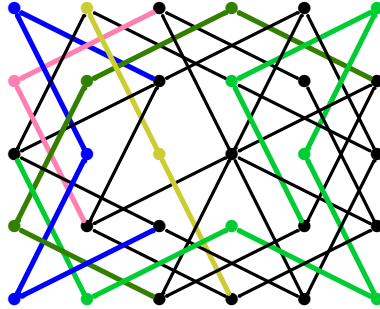
Select at the degree-2 vertices:



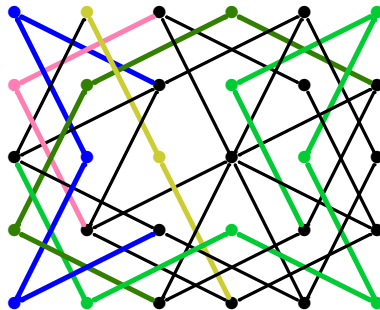
Prune:



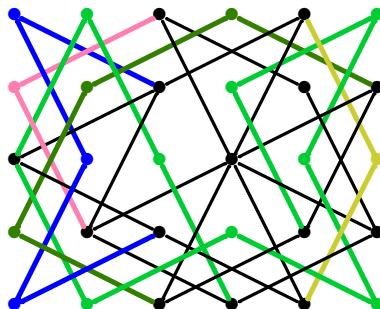
To avoid completing a dark-green 6-cycle, we must use the light green edge shown here:



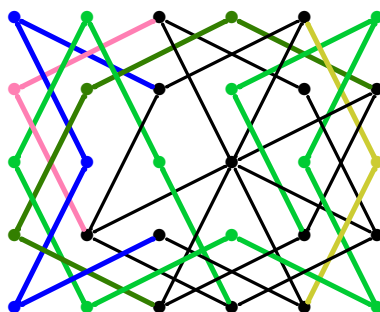
Prune:



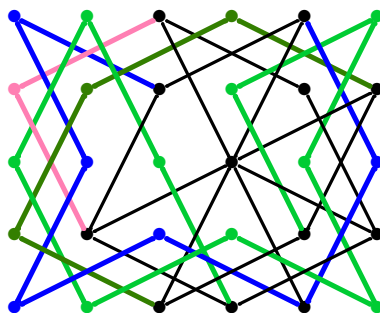
Select where the degree is 2, are recolor:



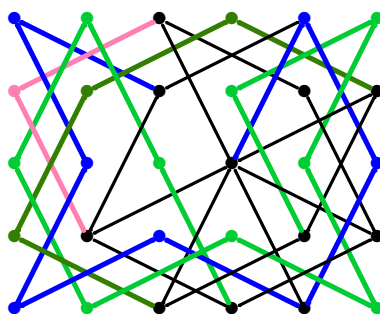
Prune:



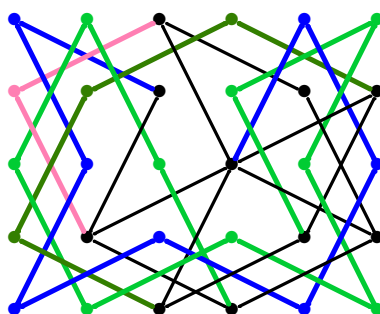
Select at degree-2, and recolor:



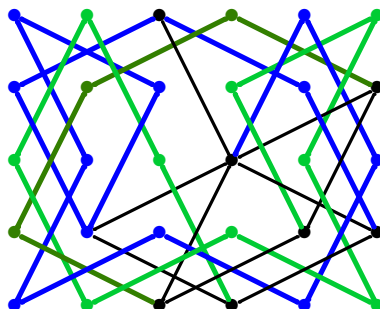
To avoid closing a blue 8-cycle, we need to select the blue edge shown:



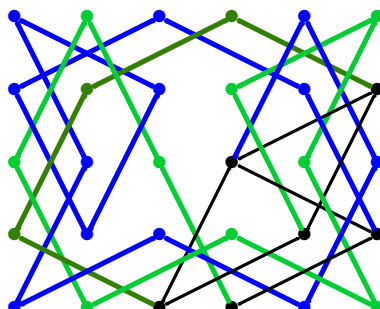
Prune:



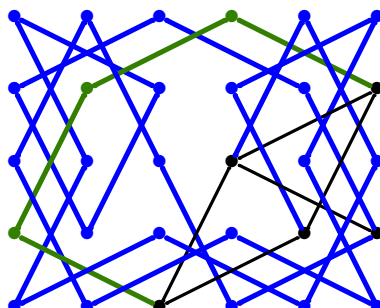
Select where the degree is 2, and recolor:



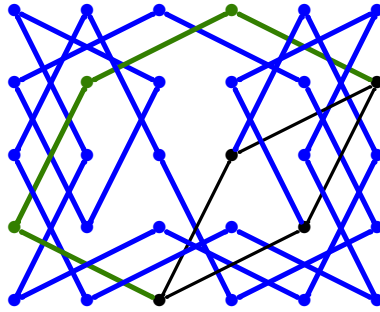
Prune:



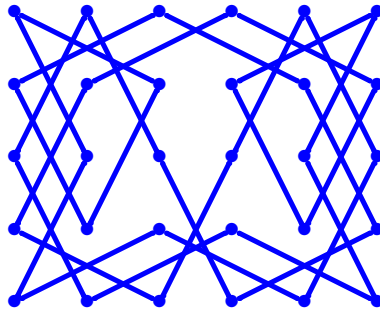
Select where the degree is 2, and recolor:



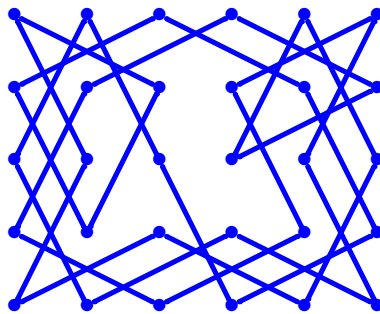
Prune:



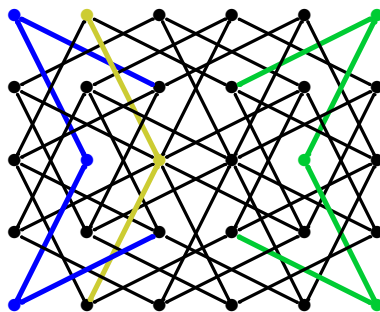
This leads to two solutions:



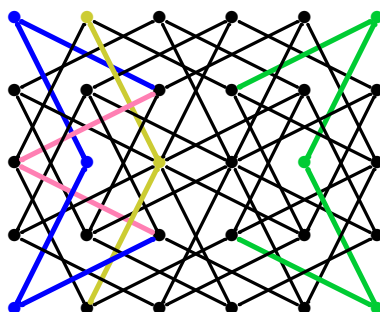
and



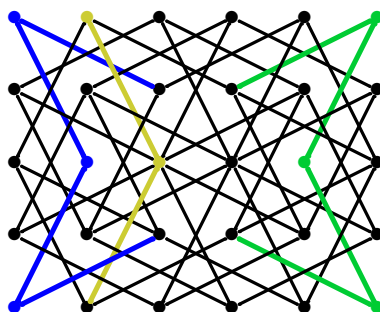
0.0.5. *Case V:*



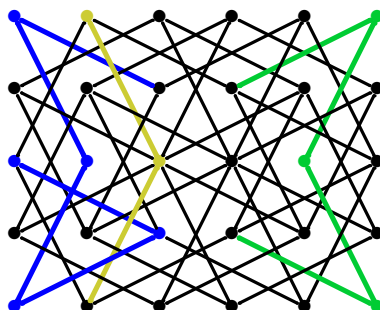
To avoid completing a blue 6-cycle, we must use one of these pink edges:



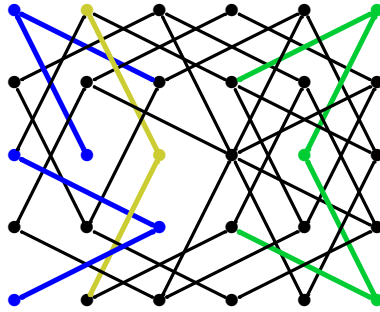
In this case, we have vertical symmetry, so without loss of generality we can remove the top one:



To avoid completing a yellow 4-cycle, we must use the blue edge shown:

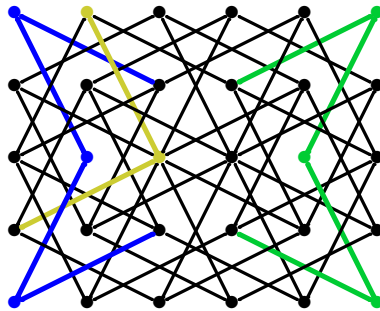


Prune:

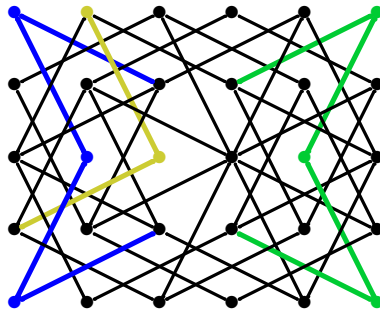


This created a vertex of degree one, so this case is impossible.

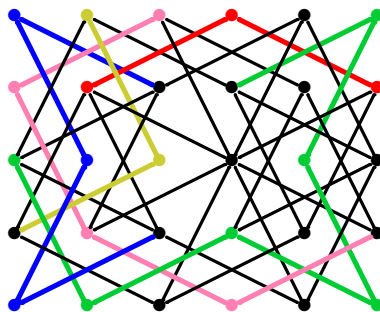
0.0.6. *Case VI.*



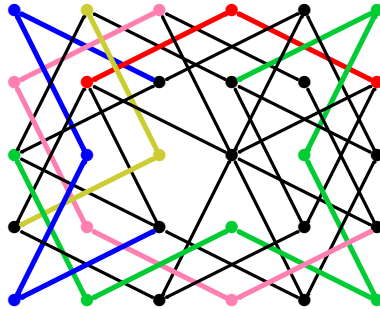
Prune:



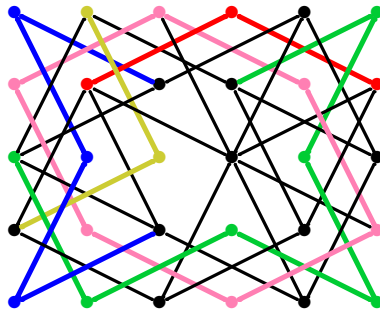
Select where the degree is 2 :



Prune:

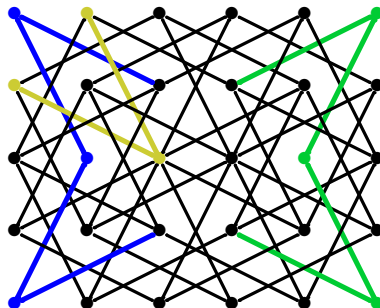


If we select where the degree is 2, we get a six cycle.

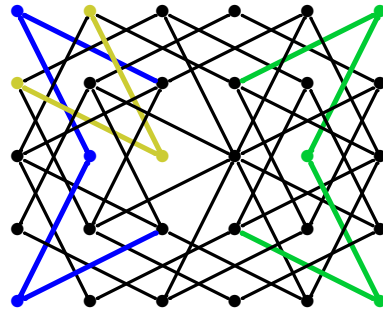


This is a dead end.

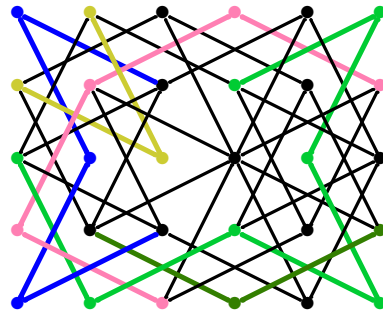
0.0.7. *Case VII.*



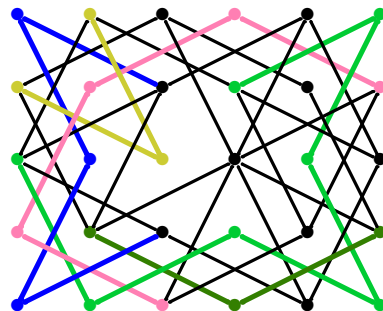
Prune:



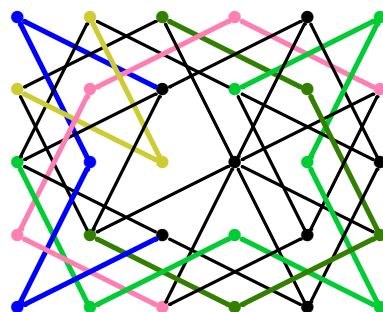
Select where the degree is 2 :



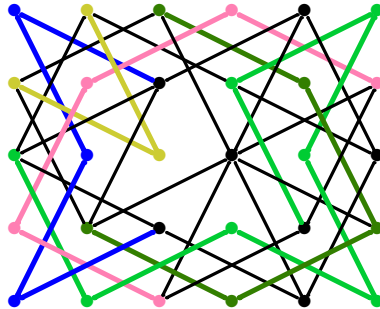
Prune:



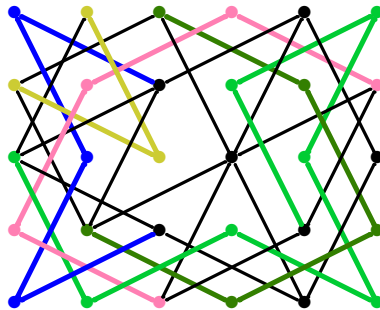
Select where the degree is 2 :



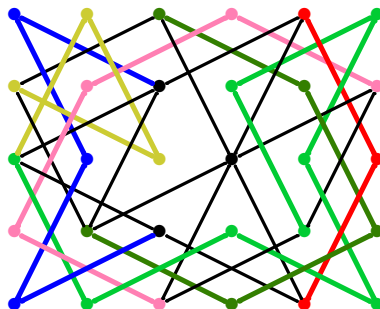
To avoid forming a light-green 6-cycle, we need to select the light green edge shown here:



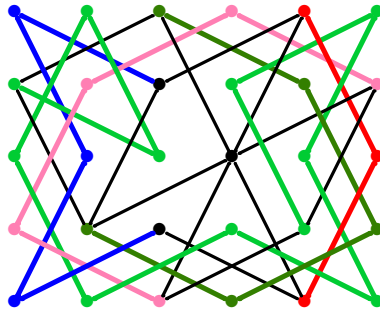
Prune:



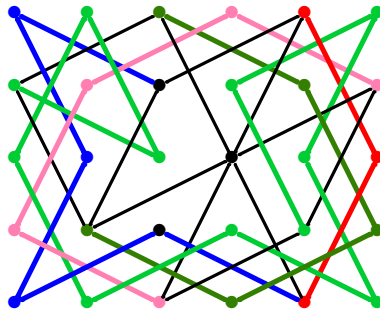
Select where the degree is 2 :



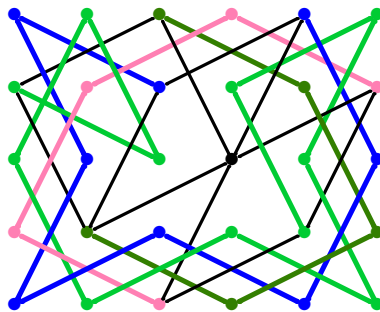
Recolor and prune:



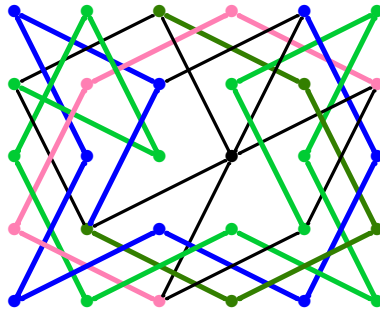
Select where the degree is 2 :



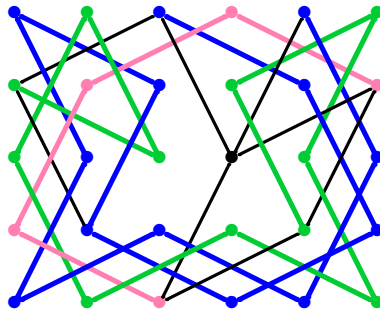
Recolor and prune:



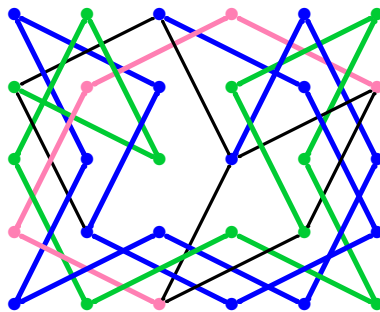
We need to need to avoid making a blue 6-cycle, so we extend the left end of the blue path downward:



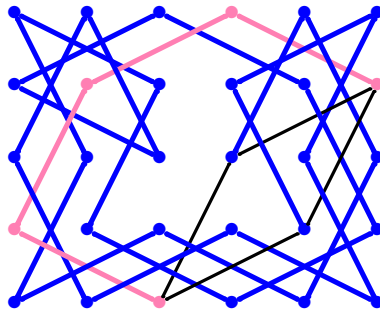
Recolor and prune:



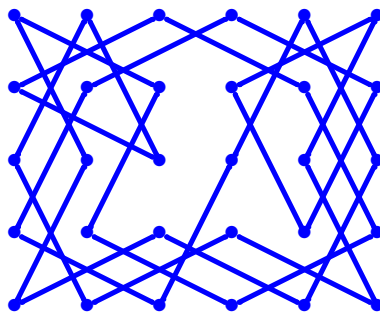
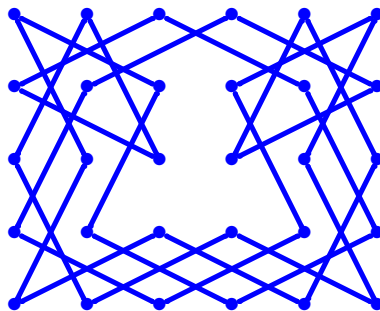
Select where the degree is 2 :



To avoid a too-small blue cycle, we need to select the edge between the blue and green paths. After recoloring and pruning, we get this:



This leads to two solutions:



0.0.8. *Conclusion:* The four Knight's tours have in some sense a duplicate, as one is the mirror image of the other. Here are the three possible Knight's tours:

