

Review Calculus

- Differentiation: Polynomial, e^x , chain rule, multiplication rule, quotient rule
- Integration: Polynomial, e^x , substitution rule, integration by parts

Differentiation

- $\frac{d}{dx}(x^n) = nx^{n-1}, \frac{d}{dx}(c) = 0$

Ex: $\frac{d}{dx}(x^3) = 3x^2$

- $\frac{d}{dx}(e^x) = e^x$

- $\frac{d}{dx}(f(x) \cdot g(x)) = f'(x)g(x) + f(x)g'(x)$

Ex: $\frac{d}{dx}(x^2 e^x) = 2x \cdot e^x + x^2 \cdot e^x$

$$\bullet \frac{d}{dx} \left(\frac{f(x)}{g(x)} \right) = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}$$

$$\text{Ex: } \frac{d}{dx} \left(\frac{e^x}{x^2} \right) = \frac{e^x \cdot x^2 - e^x \cdot 2x}{x^4}$$

$$\bullet \frac{d}{dx} \log_a x = \frac{1}{x} \log_a e$$

$$\bullet \frac{d}{dx} \ln(x) = \frac{1}{x}$$

$$\bullet \frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$$

$$\text{Ex: } \frac{d}{dx} (e^{x^2}) = e^{x^2} \cdot 2x$$

Integration

$$\int f(x) = F(x) + C, F'(x) = f(x)$$

$$\int_a^b f(x) = F(x)|_a^b, F'(x) = f(x)$$

- Polynomial $\int x^n dx = \frac{x^{n+1}}{n+1} + C$

Ex:

$$\begin{aligned}\int_0^1 x^5 dx &= \left. \frac{x^6}{6} \right|_0^1 \\ &= \frac{1}{6} - 0 \\ &= \frac{1}{6}\end{aligned}$$

- $\int e^x dx = e^x + C$

- Substitution $\int_a^b f(g(x))g'(x)dx = \int_{g(a)}^{g(b)} f(u)du$

Ex: $\int_0^1 (5x + 4)^3 dx$

Let $u = 5x + 4$, $du = 5dx$, $dx = \frac{du}{5}$

$$\begin{aligned}\int_0^1 (5x + 4)^3 dx &= \int_4^9 u^3 \frac{du}{5} \\ &= \frac{u^4}{4 \times 5} \Big|_4^9 \\ &= \frac{9^4}{20} - \frac{4^4}{20} \\ &\approx 1.28\end{aligned}$$

practice: (1) $\int e^{-5x} dx$

(2) $\int xe^{x^2} dx$

- Integral by parts

$$\int u dv = uv - \int v du$$

Ex: $\int xe^{-x} dx$

Let $u = x, v = -e^{-x}$

$du = dx, dv = e^{-x} dx$

$$\begin{aligned}\int xe^{-x} dx &= \int u dv \\ &= uv - \int v du \\ &= -xe^{-x} - \int -e^{-x} dx \\ &= -xe^{-x} + \int e^{-x} dx \\ &= -xe^{-x} - e^{-x} + C\end{aligned}$$