Stat 553 Hw5, Due May 5th (Tuesday) in class

Chapter 8:

1. Suppose a box contains four marbles, θ white ones and $4-\theta$ black ones. Test $H_0 : \theta = 2$ against $H_1 : \theta \neq 2$ as follows: Draw two marbles with replacement and reject H_0 if both marbles are the same color; otherwise do not reject.

(a) Compute the probability of Type I error.

(b) Compute the probability of Type II error for all possible situations.

(c) Rework (a) and (b) if the two marbles are drawn without replacement.

2. Let $X_1, X_2, \cdots, X_n$ be a random sample of size n from an exponential distribution, $X_i \sim Exp(1, \eta)$ ($x \sim exp(\theta, \eta), f(x|\theta, \eta) = \frac{1}{\theta} e^{-(x-\eta)/\theta}, \theta > 0, x > \eta$). A test of $H_0 : \eta \leq \eta_0$ versus $H_1 : \eta > \eta_0$ is desired, based on $X_{(1)}$.

(a) Find a critical region of size α of the form $(X_{(1)} \geq c)$.

(b) Derive the power function for the test of (a).

(c) Derive a formula to determine the sample size n for a test of size α with $\beta = P(TypeII)$ if $\eta = \eta_1$.

3. Let $X_1, X_2, \cdots, X_n$ be a random sample of size n from a bernoulli distribution with parameter θ. A test of $H_0 : \theta \leq .5$ versus $H_1 : \theta > .5$ is desired. Determine the LRT statistic $\lambda(\mathbf{x})$.

Also do the following from the textbook: P 403-406 8.10, 8.20, 8.22, 8.29

Chapter 9:

1. Consider a random sample of size n from an exponential distribution, $X_i \sim Exp(1, \eta)$ (note, here $\theta = 1$).

(a) Show that $Q = X_{(1)} - \eta$ is a pivotal quantity and find its distribution.

(b) Derive a 100 γ % equal tailed confidence interval for η.

(c) The following data are mileages for 19 military personnel carriers that failed in services: 162, 200, 271, 320, 393, 508, 539, 629, 706, 777, 884, 1008, 1101, 1182, 1463, 1603, 1984, 2355, 2880. Assuming that these data are observations of a random sample from an exponential distribution, find a 90% confidence interval for η. Assume that $\theta = 850$ is known.

Also, do problem 9.23 in the book P 455.