*ugds*, the number of undergraduate students at the institution. The value of $S_h$ in Table 3.8 is the population standard deviation of *ugds* for the colleges in stratum $h$.

**TABLE 3.8**
Allocations for a stratified sample of size 200 in Example 3.12.

| **Stratum** | $N_h$ | $S_h$ | **Proportional** | **Neyman** |
|---|---|---|---|---|
| Very small | 195 | 251 | 28 | 3 |
| Small, primarily nonresidential | 45 | 784 | 7 | 2 |
| Small, primarily residential | 123 | 515 | 18 | 3 |
| Small, highly residential | 347 | 508 | 51 | 10 |
| Medium, primarily nonresidential | 80 | 2490 | 12 | 11 |
| Medium, primarily residential | 160 | 2150 | 23 | 19 |
| Medium, highly residential | 158 | 1473 | 23 | 13 |
| Large, primarily nonresidential | 95 | 11273 | 14 | 59 |
| Large, primarily residential | 126 | 9178 | 18 | 64 |
| Large, highly residential | 43 | 6844 | 6 | 16 |
| Total | 1372 | | 200 | 200 |

The Neyman allocation has much larger sample sizes in the last three strata than the proportional allocation. This results in much lower variances for the estimated mean of variable *ugds*, which provided the population variances for the table. The anticipated variance for the mean of *ugds*, calculated using (3.4), is 82,937 for proportional allocation and 20,814 for Neyman allocation. Either allocation gives a huge reduction from the variance of the mean of *ugds* from an SRS of size 200, which is 270,689.

The gains in precision from the Neyman allocation are not likely to be as dramatic for variables that were not used for the optimization, since these variables have different relative values of $S_h^2$. But one would expect the Neyman allocation to reduce variances for variables that are correlated with *ugds* such as total instructional budget.

The Neyman allocation would be expected to yield higher variances than proportional allocation, however, for variables that are not related to *ugds*. Consider variable *majwomen*, which equals 1 if the majority of the undergraduate students in the college are women and 0 otherwise. The correlation between *majwomen* and *ugds* is about $-0.07$. The anticipated margin of error for estimating the percentage of colleges with *majwomen* $= 1$ under proportional allocation with $n = 200$ is about 5.1 percentage points, slightly less than the anticipated margin of error for an SRS (5.2 percentage points). But the anticipated margin of error for Neyman allocation is about 11 percentage points. For *majwomen*, proportional allocation results in a slightly lower variance than an SRS, but the variance from Neyman allocation is much higher than that from an SRS.

This example illustrates some of the trade-offs that must be considered when deciding on an allocation. An allocation that is optimal for one variable may be sub-optimal for another. For this survey, where most of the variables of interest are thought to be correlated with *ugds* but a few variables such as *majwomen* are not, one might want to consider an allocation that is between the proportional and Neyman (for *ugds*) allocations. ■

Allocation methods provide guidelines, not prescriptions. You can explore as many designs as you like before choosing one. And you may want to mix features of different allocations so that your sample will be suitable for multiple objectives. For example, in a stratified random sample of persons, you may want to assign an initial sample size of 400 persons to each stratum, so that estimates of percentages in each stratum have margin of error at most 5 percentage points, and then assign the remaining sample size proportionally or optimally to the strata.