TABLE 6.5
Inclusion probabilities (π_i) and joint inclusion probabilities (π_{ik}) for samples of size 2 that could be selected using the method in Example 6.8. The entries of the table are the π_{ik}'s for each pair of stores (rounded to four decimal places); the margins give the π_i's for the four stores

		Store k				
		A	B	C	D	π_i
Store i	A	—	0.0173	0.0269	0.1458	0.1900
	B	0.0173	—	0.0556	0.2976	0.3705
	C	0.0269	0.0556	—	0.4567	0.5393
	D	0.1458	0.2976	0.4567	—	0.9002
	π_k	0.1900	0.3705	0.5393	0.9002	2.0000

size 2 consists of psus i and k:

$$\text{For } n = 2,\quad P(\text{units } i \text{ and } k \text{ in sample}) = \pi_{ik} = \psi_i \frac{\psi_k}{1-\psi_i} + \psi_k \frac{\psi_i}{1-\psi_k}.$$

The probability that psu i is in the sample is then

$$\pi_i = \sum_{\mathcal{S}:\, i \in \mathcal{S}} P(\mathcal{S}).$$

Table 6.5 gives the π_i's and π_{ik}'s for the supermarkets. ■

6.4.1 The Horvitz–Thompson Estimator for One-Stage Sampling

Assume we have a without-replacement sample of n psus, and we know the **inclusion probability**

$$\pi_i = P(\text{unit } i \text{ in sample})$$

and the **joint inclusion probability**

$$\pi_{ik} = P(\text{units } i \text{ and } k \text{ are both in the sample}).$$

The inclusion probability π_i can be calculated as the sum of the probabilities of all samples containing the ith unit and has the property that

$$\sum_{i=1}^{N} \pi_i = n. \tag{6.17}$$

For the π_{ik}'s, as shown in Theorem 6.1 of Section 6.6,

$$\sum_{\substack{k=1\\k\neq i}}^{N} \pi_{ik} = (n-1)\pi_i. \tag{6.18}$$