Stat 476/576:

Assignment 1: Due Feb 5 Tuesday in class

Instruction: For problem 1 to problem 6, do not use software, write steps how you get the solution.

Problem 1. Define matrices $\mathbf{A}, \mathbf{B}, \mathbf{C}$ and $\mathbf{D}$ as follows:

$$\mathbf{A} = \begin{bmatrix} 2 & 4 & 0 \\ -3 & 2 & 5 \\ 1 & 3 & -2 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 4 & -3 & 2 \\ 3 & 5 & 4 \\ -4 & 2 & 5 \end{bmatrix}, \mathbf{C} = \begin{bmatrix} 2 & 4 \\ 3 & 2 \\ 0 & 5 \end{bmatrix}, \mathbf{D} = \begin{bmatrix} 2 & 0 \\ 4 & 5 \end{bmatrix},$$

Find:

(a) $\mathbf{A} + \mathbf{B}$ (b) $2\mathbf{A} + \mathbf{B}$ (c) $\mathbf{A}' - \mathbf{B}'$

(d) $\mathbf{AC}$ (e) $\mathbf{DC}'$

(f) $\mathbf{D}^{-1}$ (g) $|A|$ (h) Rank of $\mathbf{A}$

Problem 2. Are

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$$

linearly independent or not? (Be explicit)

Problem 3. Given the matrix $\mathbf{A}$ below, find the eigenvalues and associated normalized eigenvectors. Determine the spectral decomposition of $\mathbf{A}$.

$$\mathbf{A} = \begin{bmatrix} 5 & -3 \\ -3 & 5 \end{bmatrix}$$

Problem 4. Exercise 1.3 on page 38 of J & W

Problem 5. Exercise 2.5 on page 103 of J & W

Problem 6. Exercise 2.6 on page 103 of J & W

Problem 7. Use R for this problem, (**For all problems using R, please attach R code as part of your homework**)

Create the following matrix in R

$$\mathbf{X} = \begin{bmatrix} 10 & 4 & 2 \\ 4 & 3 & 0 \\ 2 & 0 & 5 \end{bmatrix}$$

(a) Does this matrix have an inverse? If so, find it.

(b) What are the eigenvectors and eigenvalues of $\mathbf{X}$?

(c) What is $\mathbf{X}^{1/2}$?

(d) What is $\mathbf{X}^{-1/2}$?

(d) Find a generalized inverse of $\mathbf{X}$.

Assignment 2: Due Feb 19 Tuesday in class

Problem 1. Let $\mathbf{X} = (X_1, X_2, X_3)' \sim N_n(\mathbf{u}, \mathbf{\Sigma})$, where

$$\mathbf{u} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad \text{and} \quad \mathbf{\Sigma} = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}.$$

(a) What is the marginal distribution of X_3?

(b) Write the marginal distribution of $(X_1, X_2)'$

(c) Find the conditional distribution of X_3 given $(X_1, X_2)'$. Explain why this variance is smaller than the variance found in part (a) for the marginal distribution of X_3.

(d) Find $P(X_3 > 1 | X_1 = 2.5, X_2 = 2.5)$

(e) Find the conditional distribution of X_1 and X_3 given $X_2 = x_2$

(f) What is the correlation coefficient between X_1 and X_3 given $X_2 = x_2$. This is known as the partial correlation coefficient.

Problem 2. Exercise 4.4 on page 201 of J & W. Part (a) only.

(b) What is the distribution of

$$\mathbf{Y} = \begin{bmatrix} 3 & 1 & 2 \\ 2 & 3 & -1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$$

Problem 3. Exercise 4.18 on page 207 of J & W.

Problem 4. Exercise 4.22 on page 207 of J & W.

Problem 5. Exercise 4.39 on page 207 of J & W, but answer the following questions instead of those in textbook

(a) Examine each of the five variables for marginal normality. Comment.

(b) Examine all pairwise scatterplots for bivariate normality. Comment.

(c) Calculate the statistical distance from each observation to the mean. Comment on any outliers.

(d) Are transformations necessary to any of the variables to help satisfy the normality assumption? If so, apply appropriate transformations and verify that the normality assumption is feasible for the transformed data.

Assignment 3: Not collected

Problem 1. Let $\mathbf{X} = (X_1, X_2, X_3, X_4)'$ be distributed as $N(\mathbf{u}, \mathbf{\Sigma})$. We wish to test $H_0 : \mathbf{u}_1 = \mathbf{u}_2$ and $\mathbf{u}_3 = \mathbf{u}_4$ versus $H_\alpha : \mathbf{u}_1 \neq \mathbf{u}_2$ or $\mathbf{u}_3 \neq \mathbf{u}_4$.

(a) Construct a test statistic for testing H_0 versus H_α.

(b) Based on a sample of size $n = 20$, what values of your test statistic would lead you to reject H_0? Use $\alpha = 0.05$.

Problem 2. Use the data on Biochemical compound below to test $H_0 : \mathbf{u}_c = \mathbf{u}_t$ v.s. $H_\alpha : \mathbf{u}_c \neq \mathbf{u}_t$ using Hotelling's T^2. Assume that $\mathbf{\Sigma}_1 = \mathbf{\Sigma}_2$. Complete other assumptions for the test.

Control Group										
x_1	1.21	0.92	0.80	0.85	0.98	1.15	1.10	1.02	1.18	1.09
x_2	0.61	0.43	0.35	0.48	0.42	0.52	0.50	0.53	0.45	0.40

Control Group												
x_1	1.40	1.17	2.23	1.19	1.38	1.17	1.31	1.30	1.22	1.00	1.12	1.09
x_2	0.50	0.39	0.44	0.37	0.42	0.45	0.41	0.47	0.29	0.30	0.27	0.35

Problem 3. Exercise 5.1 on page 261 of J & W.

Problem 4. Exercise 5.7 on page 261 of J & W.

Assignment 4: Due 04/09/2013 Tuesday

Page 470 Chapter 8, 8.1, 8.12, 8.13

Assignment 5: Due 04/23/2013 Tuesday

Problem 1. Chapter 9, 9.28

Problem 2. Suppose we have a sample from population π_1 and a sample from population π_2.

From π_1:

$$\begin{pmatrix} 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 5 \\ 7 \end{pmatrix}, \begin{pmatrix} 5 \\ 4 \end{pmatrix}, \begin{pmatrix} 8 \\ 7 \end{pmatrix}, \begin{pmatrix} 5 \\ 8 \end{pmatrix}, \begin{pmatrix} 4 \\ 6 \end{pmatrix}$$

From π_2:

$$\begin{pmatrix} 12 \\ 9 \end{pmatrix}, \begin{pmatrix} 16 \\ 9 \end{pmatrix}, \begin{pmatrix} 14 \\ 8 \end{pmatrix}, \begin{pmatrix} 10 \\ 8 \end{pmatrix}, \begin{pmatrix} 12 \\ 8 \end{pmatrix}, \begin{pmatrix} 14 \\ 12 \end{pmatrix}$$

(a) Find the sample classification function and classification rule.

(1) using equal prior probabilities.

(2) using $p_1 = .6$ and $p_2 = .4$ as prior probabilities.

(b) Plot the data points and the two lines given by the two classification rules in part (a). Use different plot symbols to indicate from which sample each observation comes.

(c) Find the classification scores for the following observations and classify them in the equal prior case and the $p_1 = .6$ and $p_2 = .4$ case. Also plot these new observations on the graph from (b) –use a different symbol for these points.

$$\begin{pmatrix} 9 \\ 9 \end{pmatrix}, \begin{pmatrix} 14 \\ 11 \end{pmatrix}, \begin{pmatrix} 8 \\ 7 \end{pmatrix}, \begin{pmatrix} 7 \\ 6 \end{pmatrix}, \begin{pmatrix} 10 \\ 9 \end{pmatrix}$$

(d) Compute D^2-the sample Mahalanobis distance between the two sample mean vectors.

(e) Estimate $P(1|2)$ and $P(2|1)$ for the equal prior case.

(f) Calculate the posterior probability for each observation in part (c) in the equal prior case, and use this probability to classify each of the observations.