8/25/2010. Thursday Time series. ①

white noise: The time series generated from uncorrelated variables is used as a model for noise in engineering application, where it is called white noise.

white noise

$W_t \sim wn(0, 6w^2)$

the designation white originates from the analogy with white light & indicates that all possible periodic oscillations are present with equal strength.

white independent noise: require the noise to be iid random variables. with mean 0 & variance $6w^2$. $W_t \sim iid(0, 6w^2)$

Gaussian white noise: Wt are independent ~~white noise series~~ normal random variables. with mean 0 & variance 6_w^2.

$W_t \overset{iid}{\sim} N(0, 6w^2)$

Example 1.9. Moving average.

replace W_t by an average of its current value and its immediate neighbors in the past and future.

Let $V_t = \frac{1}{3}(W_{t-1} + W_t + W_{t+1})$.

expect a smoother version of the first white noise series. the slower oscillations are more apparent & some of the faster oscillations are taken out.

rnorm (random variable)

filter: linear filtering on a T.S.
filter(x, filter, method = c("convolution", "recursive"), sides = 2, circular = False, init)

par(mfrow = c(2,2)) # 2x2 picture on one plot.
mfrow = c(2,1) # 2x1 pictures on one plot.

plot.ts
plot time series

Sides = 1. the fileer coefficiene are for pase values only.

Sides = 2. they are cenered around lag 0. length of the fileer should be odd, bue if it is even, more of the fileer is forward in time than backward.

Random Walk.

Example 1.11.

$$x_t = \delta + x_{t-1} + w_t \quad * \qquad \text{noise.}$$

$t = 1, 2 \ldots$ initial condition $x_0 = 0$. w_t white

δ : drift. $\delta = 0$ $x_t = x_{t-1} + w_t$ is called a random walk.

The term random walk comes from the fact that, when $\delta = 0$, the value of the time series at time t is the value of the series at time $t-1$ plus a completely random movement determined by w_t.

using induction.

$X_t = \delta + X_{t-1} + W_t$.

$X_0 = 0$.

$X_1 = \delta + 0 + W_1$

$X_2 = \delta + X_1 + W_2 = \delta + \delta + W_1 + W_2$.

$= 2\delta + W_1 + W_2$.

$X_3 = \delta + X_2 + W_3$

$= \delta + 2\delta + W_1 + W_2 + W_3$

$= 3\delta + \sum_{i=1}^{3} W_i$

$\vdots$

$\therefore X_t = \delta t + \sum_{j=1}^{t} W_j$. $t = 1, 2 \cdots$

R Code:

```
>set.seed (154)
>W<-rnorm (200,0,1);  X= cumsum(w)
>wd = w+.2;   Xd = cumsum(wd).
>plot.ts (xd, ylim = c(-5,55))
>lines (X)
>lines (.2*(1:200), lty= "dashed")
```

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Example 1.17. Auto covariance of a Moving average. $W_t \sim (0, \sigma_w^2 = 1)$.

$$V_t = \frac{1}{3}(W_{t-1} + W_t + W_{t-1})$$

$$E(V_t) = 0.$$

$$\gamma_v(s,t) = E[(V_s - 0)(V_t - 0)]$$

$$= \frac{1}{9} E[(W_{s-1} + W_s + W_{s+1})(W_{t-1} + W_t + W_{t+1})]$$

Let $s - t = h$.

$h = 0, \pm 1, \pm 2 \ldots$

$h = 0$.

$$\gamma_v(t,t) = \frac{1}{9} E[(W_{t-1} + W_t + W_{t+1})(W_{t-1} + W_t + W_{t+1})]$$

$$= \frac{1}{9}[E(W_{t-1}W_{t-1}) + E(W_t W_t) + E(W_{t+1}W_{t+1})]$$

$$= \frac{3}{9}.$$

$E(X) = 0$

$V(X) = 1$

$V(X) = E(X^2) - [E(X)]^2$

$\therefore E(X^2) = 1$

$h = 1.$

$$\gamma_v(t+1, t) = \frac{1}{9}E[(w_t + w_{t+1} + w_{t+2})(w_{t-1} + w_t + w_{t+1})]$$
$$= \frac{1}{9}E[(w_t w_t) + E(w_{t+1} w_{t+1})]$$
$$= \frac{2}{9}.$$

Similar computations give $\gamma_v(t-1, t) = 2/9$

$$\gamma_v(t+2, t) = \gamma_v(t-2, t) = 1/9.$$

$$\therefore \gamma_v(s,t) = \begin{cases} \frac{3}{9} & s = t \\ 2/9 & |s-t| = 1 \\ 1/9 & |s-t| = 2 \\ 0 & |s-t| \geq 3 \end{cases}$$

① Shows that the smoothing operation introduces a covariance function that decreases as the separation between the two time points increases and disappears completely when the time points are separated by three or more time points.

② depends on the time separation or lag and not on the absolute location of the points along with data

Example 1.18. Autocovariance of a random walk.

$x_t = \sum_{j=1}^{t} w_j$. we have.

$\gamma_x(s,t) = Cov(x_s, x_t) = Cov\left(\sum_{j=1}^{s} w_j, \sum_{k=1}^{t} w_k\right)$

$= \min\{s, t\}\, \sigma_w^2$

w_t uncorrelated ~~ran~~ r.v.s.

① autocovariance function of a random walk depends on the time values s, t. not on the time separation or lag. ↑

② $Var(x_t) = \gamma_x(t,t) = t\,\sigma_w^2$. ↑ without bound as time t ↑.