9/2/2010 Thursday Stat 681/581.

Def 1.4. The cross-covariance function between two series x_t & y_t is

$$\gamma_{xy}(s,t) = E[(x_s - \mu_{xs})(y_t - \mu_{yt})]$$

Def 1.5. The cross-correlation function (CCF)

$$\rho_{xy}(s,t) = \frac{\gamma_{xy}(s,t)}{\sqrt{\gamma_x(s,s)\,\gamma_y(t,t)}}$$

For stationary data

Def 1.10. Two time series, say x_t & y_t, are said to be jointly stationary if they are each stationary, and the cross-covariance function.

$$\gamma_{xy}(h) = E[(x_{t+h} - \mu_x)(y_t - \mu_y)]$$
$$= f(|h|)$$

Def 1.11. The cross-correlation function (CCF) of jointly seationary time series X_t & Y_t is defined as ②

$$\rho_{xy}(h) = \frac{\gamma_{xy}(h)}{\sqrt{\gamma_x(0)\gamma_y(0)}} \qquad -1 \le \rho_{xy}(x) \le 1$$

when looking at the relation between X_{t+h} & Y_t, the cross-correlation function satisfies $\rho_{xy}(h) = \rho_{yx}(-h)$

Example 1.21, joint seationary

$X_t = W_t + W_{t-1}$ sum of two successive white noise

$Y_t = W_t - W_{t-1}$ difference of two successive white noise

$W_t \overset{\text{independent}}{\sim} (0, \sigma_w^2)$

① each seationary

$E(X_t) = 0. \quad E(Y_t) = 0$

$$\gamma_x(s,t) = E[(X_s - 0)(X_t - 0)]$$
$$= E[(W_s + W_{s-1})(W_t - W_{t-1})]$$

Let $s-t$, $s=t+h$

$\gamma_x(s,t) = E[(w_{t+h} + w_{t+h-1})(w_t - w_{t-1})]$

$= \begin{cases} 2\sigma_w^2 & h=0 \\ \sigma_w^2 & h=\pm 1 \\ 0 & |h| \geq 2 \end{cases}$

Similarly

$\gamma_y(s,t) = \begin{cases} 2\sigma_w^2 & h=0 \\ -\sigma_w^2 & h=\pm 1 \\ 0 & |h| \geq 2. \end{cases}$

$\rho_{xy}(h) = \dfrac{\gamma_{xy}(h)}{\sqrt{\gamma_x(0)\gamma_y(0)}}$

$\gamma_{xy}(1) = E[(x_{t+1} - u_x)(y_t - u_y)]$

$= E[(w_{t+1} + w_t)(w_t - w_{t-1})]$

$= E[w_t^2] = \sigma_w^2$

$\gamma_{xy}(0) = 0.$ $\gamma_{xy}(-1) = -\sigma_w^2$. $|h| \geq 2$. $\gamma_{xy}(h) = 0$

$\therefore \rho_{xy}(h) = \begin{cases} 0 & h=0 \\ 1/2 & h=1 \\ -1/2 & h=-1 \\ 0 & |h| \geq 2 \end{cases}$

CCF depends only on h, the series are jointly seatronary.

Review chap 1

① Motivation.

② Definition of T.S: A stochastic process or a sequence of random variables $\{X_t; t \in S\}$. where S is some set of indices.

③ Difference between T.S and traditional Statistical Inference.

④ Main goal: $\begin{cases} E(X_t).\ V(X_t).\ Cov(X_t, X_s) \\ \text{periodicity or cycles} \\ \text{model process. forecasting} \end{cases}$

⑤ Stationarity.

⑥ Measures $\begin{cases} ACF \\ CCF \end{cases}$

⑦ Estimation of correlations

Example 1.5 from book.

using ASTSA

please download a copy of ASTSA to your own computer and begin to use it or R to do homework.