Review

- Time series (T.S): A stochastic process or a sequence of random variables $X_t, t \in S$; where S is some set of indices.

- Difference with traditional Statistical Inference: The data is assumed to be an i.i.d process (random sample). In T.S. we are relaxing this assumption and wish to model the dependency among observations.

- Second order stationarity: The mean is constant in time and the covariance is a function of the difference in time between observations.

$$E(X_t) = \mu; Cov(X_t; X_s) = f(|t-s|)$$

- Autocovariance function: For a second order stationary process, this is defined as

$$\gamma(h) = Cov(X_t; X_{t+h}) = E[(X_t - \mu)(X_{t+h} - \mu)]$$

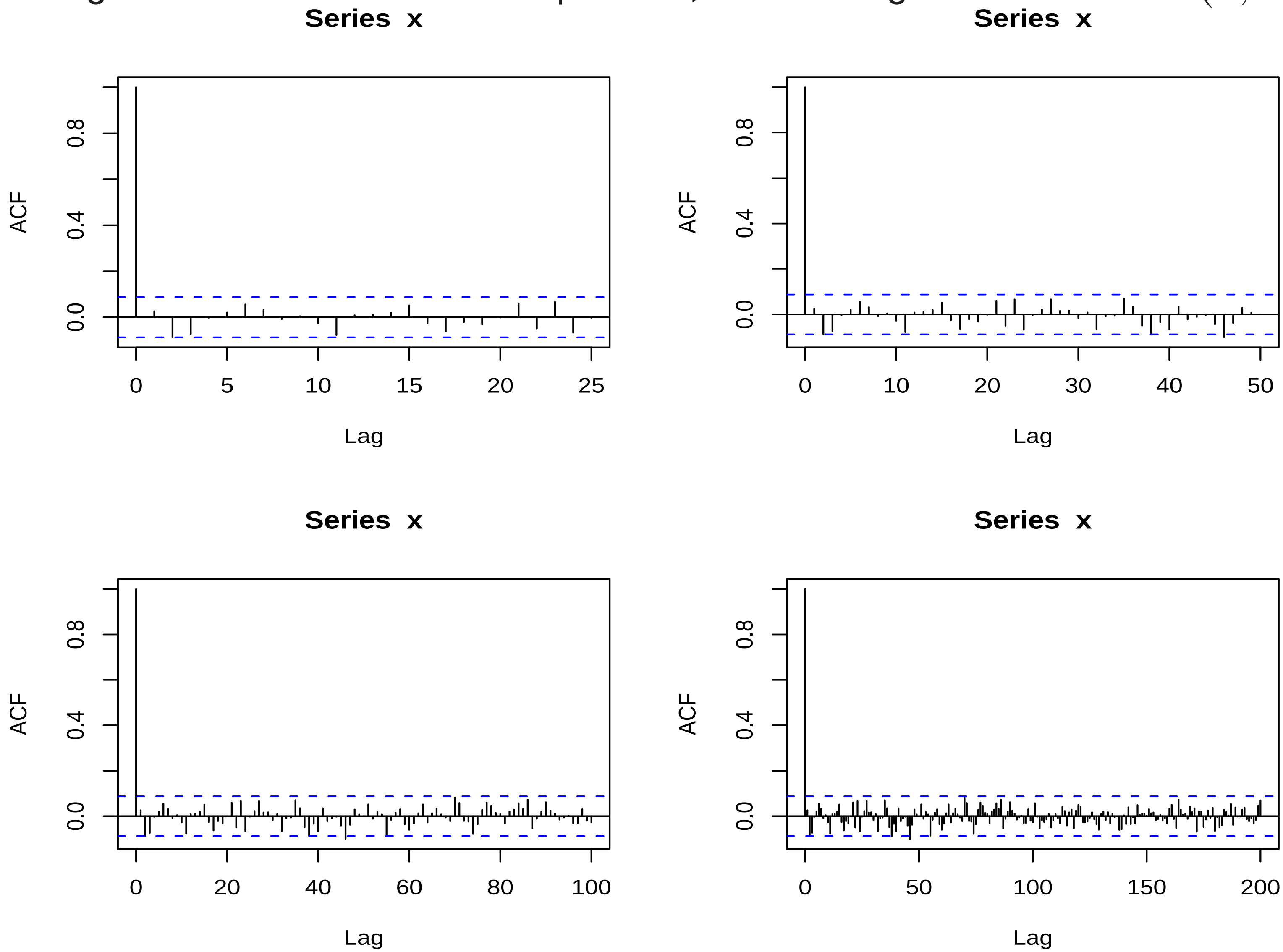
$$Cov(X_t; X_{t+h}) = Cov(X_t; X_{t-h}) = \gamma(h)$$

- Autocorrelation function of X_t is defined in terms of the autocovariance as: $\rho(h) = \gamma(h)/\gamma_0, -1 \leq \rho \leq 1$.

- A correlogram is a plot of h (x-axis) versus its corresponding value of r_h (y-axis). The correlogram may exhibit patterns and different degrees of dependency in a time series.

Example: ACF for white noise process.

Figure 1: ACF for white noise process, data were generated from $N(0,1)$



Example: ACF for 400 EEG data. Observations from an electroencephalogram corresponding to a patient undergoing ECT therapy. Data we use here are 400 observations from the central part of the series.

Figure 2: Plot of 400 EEG data

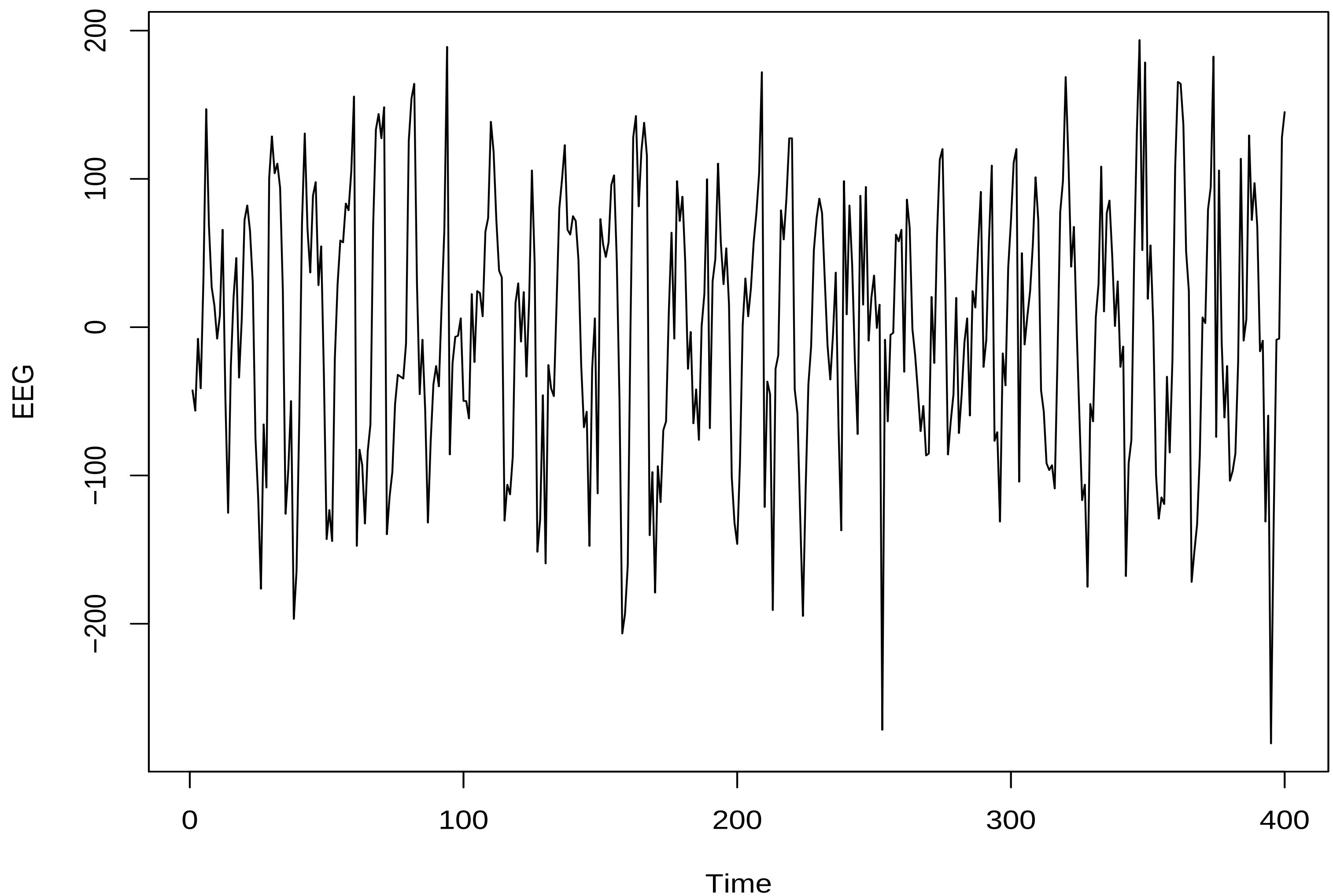
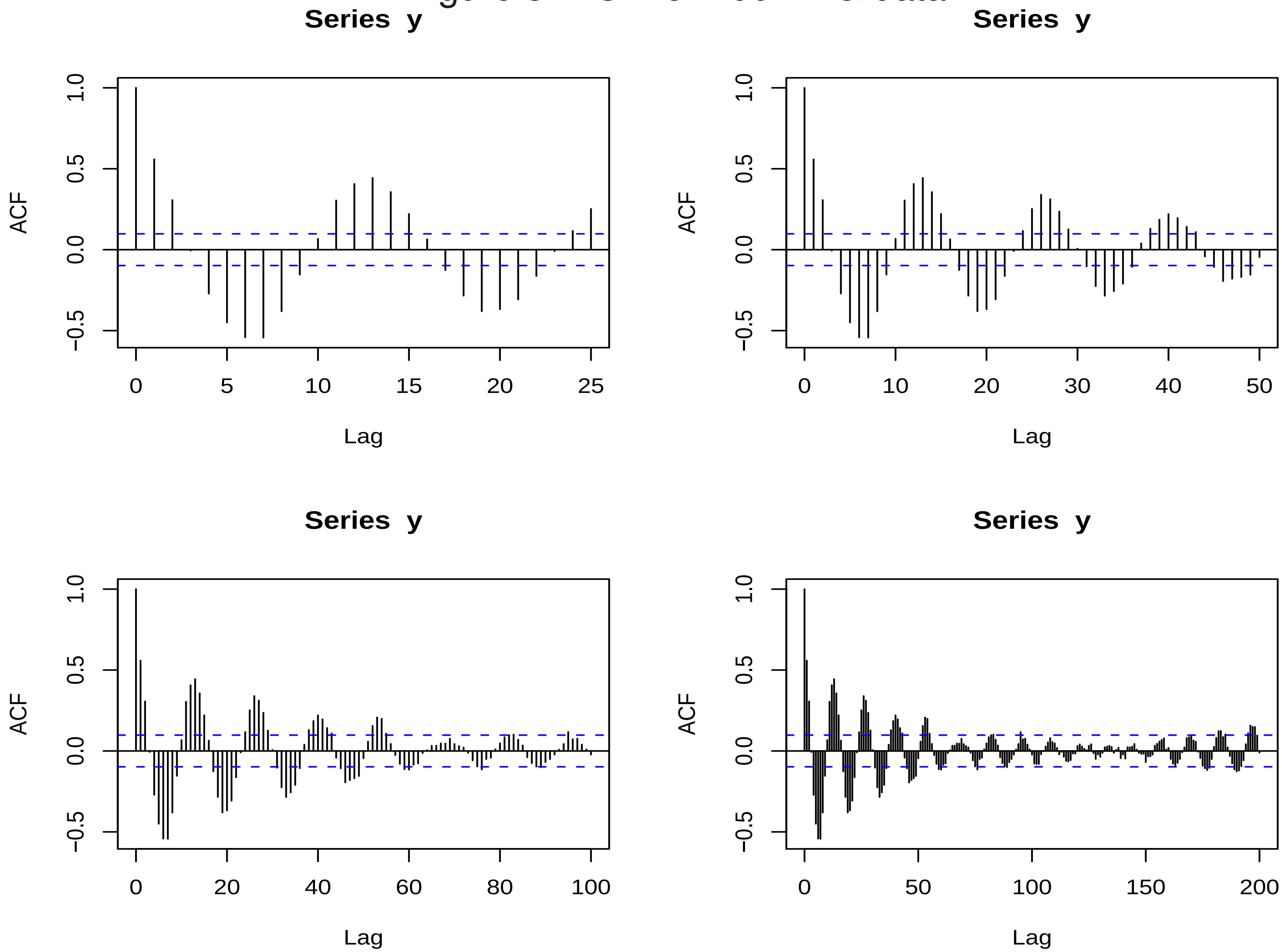


Figure 3: ACF for 400 EEG data



Example: ACF for MA(2) data. Data were generated from $X_t = w_t + .5w_{t-1} - .5w_{t-2}$

Figure 4: Plot of MA(2)

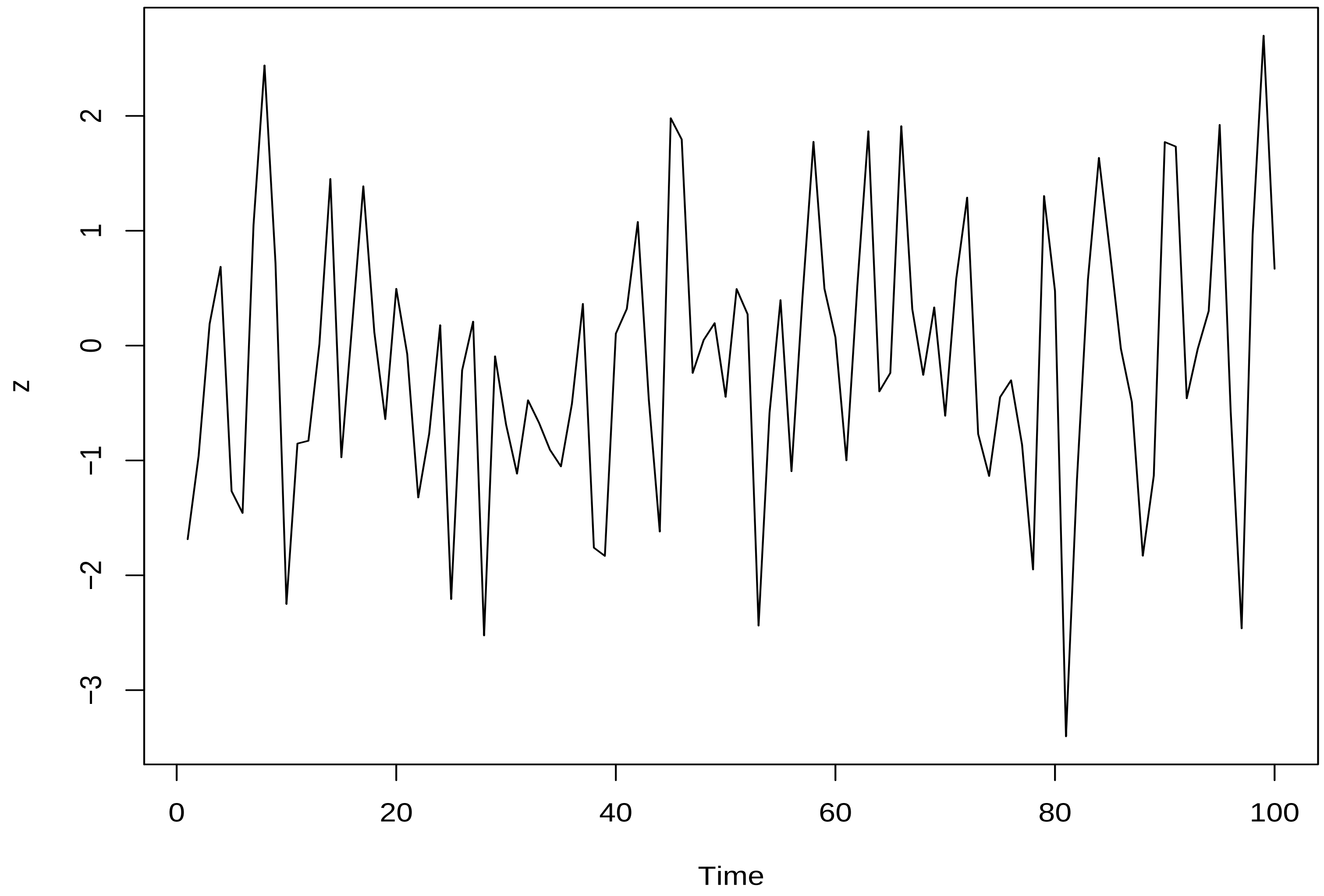
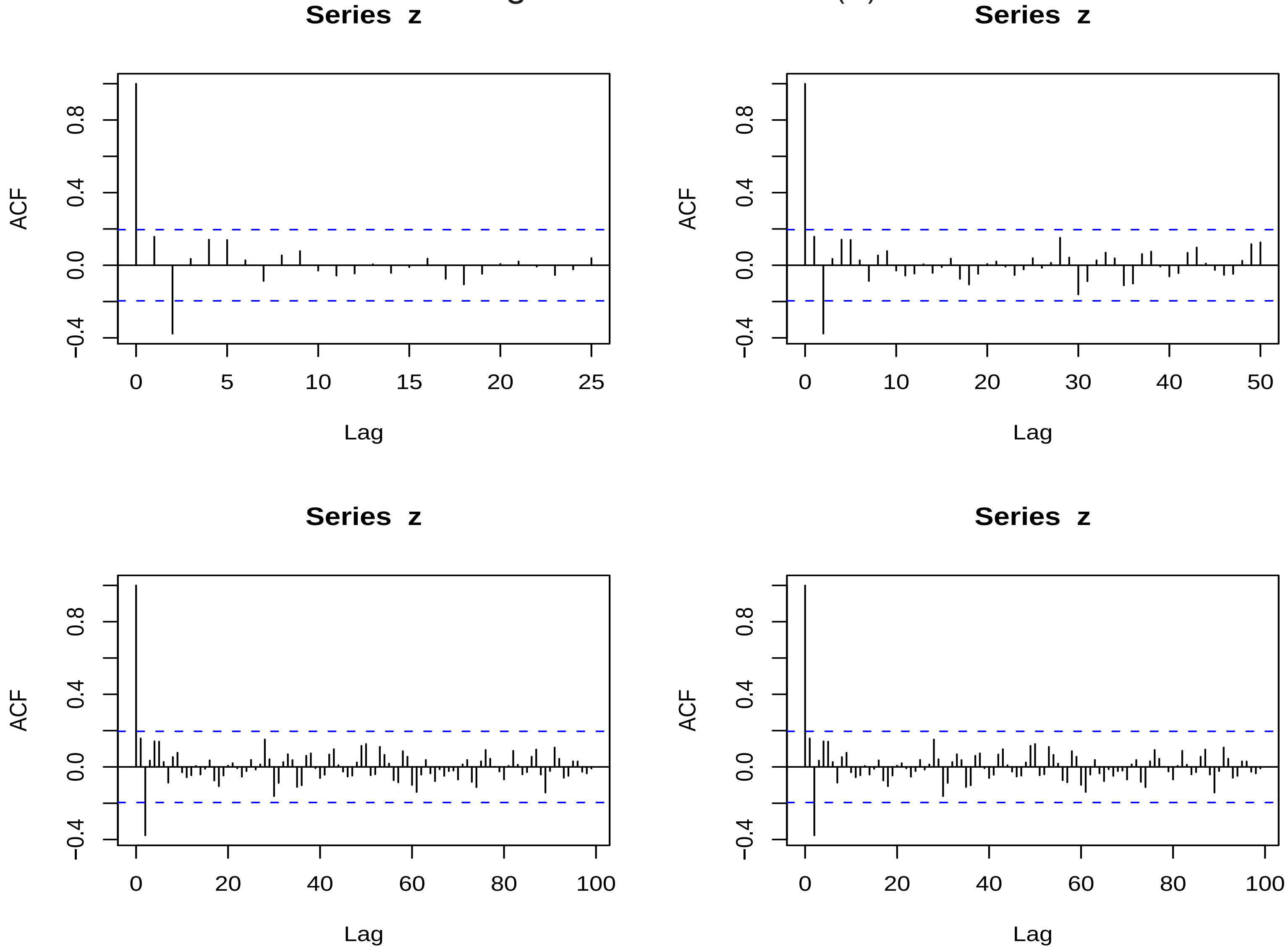


Figure 5: ACF for MA(2)



- Seasonal variation: Variation in the data that is annual in period or variation that is quarterly, monthly, weekly, etc. in period. For example, unemployment is typically 'high' in the winter but lower in the summer. If seasonality or cyclic variation are not of interest, they could be removed from the process.

- Detrend method: lowess, filtering, differencing, kernel smoothing and smoothing splines etc,. General setup $x_t = f_t + y_t$ where f_t is some smooth function of time, and y_t is a stationary process.

- AR, MA and ARMA models

Definition, Characteristic polynomial, roots, Theorem to determine the stationarity, invertible

- PACF, use cramer's rule to find the PACF

  Behavior of the ACF and PACF for stationary and invertible ARMA models

	AR(p)	MA(q)	ARMA (p,q)
ACF	Tails off	Cuts off after lag q	Tails off
PACF	Cuts off after lag p	Tails off	Tails off