Stat 481/581:

Assignment 3: Due Oct 5

1. Using the 400 observations corresponding to the "short and central" EEG series compute the periodogram and produce a plot of the frequencies versus the values of the periodogram.

2. For each of the following represenstations, check if the process X_t is stationary and/or invertible. Justify your answer in each case.

$$(1 - B)X_t = (1 - 1.5B)w_t$$

$$(1 - .8B)X_t = (1 - .5B)w_t$$

$$(1 - 1.1B + .8B^2)X_t = (1 - 1.7B + .72B^2)$$

3. For an $AR(2)$ process $X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + w_t$ with real roots, check that the condition for stationarity $(-1 < a_1 \leq a_2 < 1)$ implies that $\phi_1 + \phi_2 < 1$ and $\phi_2 - \phi_1 < 1$. Also show that the partial autocorrelation for this process has the form $P_1 = \phi_1/(1 - \phi_2)$; $P_2 = \phi_2$; $P_k = 0$ with $k \geq 3$.

4. Suppose X_t follows an MA process of order 2. That is $X_t = w_t - \theta_1 w_{t-1} - \theta_2 w_{t-2}$. Find the theoretical autocorrelation function of the process.

5. Generate 500 observations from a stationary $AR(4)$ process with two complex pairs of conjugate reciprocal roots that have the following conditions. The first pair has modulus $r_1 = .9$ and frequency $w_1 = .5$; the second pair has modulus $r_2 = 0.75$ and frequency $w_2 = 1.35$. Plot the simulated process and also the corresponding ACF and PACF.

6. Consider the infinite order MA process defined by $X_t = w_t + C(w_{t-1} + w_{t-2} + \cdots)$, where C is a constant and the w_ts are iid $N(0, \sigma^2)$ random variables.

(a) Show that the process X_t is non-stationary.

(b) Consider the series of first differences $Y_t = X_t - X_{t-1}$. Show that Y_t is a first order MA process $(MA(1))$.

(c) Find the range of values for C for which this MA process is invertible .

(d) Find the autocorrelation function of Y_t.

7. Suppose that we generated 400 observations of a time series process. The first 3 sample autocorrelations $(r(k))$ are $r(1) = 0.06, r(2) = 0.18, r(3) = .0.22$. Do we have evidence

that this simulated data does not follow a white noise process?

8. For the following statements answer TRUE or FALSE and justify your answer completely. Provide examples or counterexamples as you see fit.

(a) A time series process with a constant variance is always a second order stationary process.

(b) A time series data looks periodic and with an increasing mean level. The model Xt = a+bt+.t always gives an adequate representation to the trend of the data.

(c) Before applying any time series method, we should always take a first difference of the data. This assures stationarity of the process and then we can apply any analysis we desire, like estimating a periodogram or fitting an AR model.