prove of (2.8.49)

$$Var(\bar{X}_n) = Var\left(\frac{\sum_{i=1}^{N} z_i v_i}{n}\right)$$

$$= \frac{1}{n^2} Cov\left(\sum_{i=1}^{N} z_i v_i, \sum_{j=1}^{N} z_j v_j\right)$$

$$= \frac{1}{n^2}\left[\sum_{i=1}^{N} v_i^2 V(z_i) + \sum_{i=1}^{N}\sum_{j\neq i}^{N} v_i v_j Cov(z_i, z_j)\right]$$

$$= \frac{1}{n^2}\left[\frac{n}{N}\left(1-\frac{n}{N}\right)\sum_{i=1}^{N} v_i^2 - \sum_{i=1}^{N}\sum_{j\neq i}^{N} v_i v_j \cdot \frac{1}{N-1}\left(1-\frac{n}{N}\right)\left(\frac{n}{N}\right)\right]$$

$$= \frac{1}{n^2}\cdot\frac{n}{N}\left(1-\frac{n}{N}\right)\left[\sum_{i=1}^{N} v_i^2 - \frac{1}{N-1}\sum_{i=1}^{N}\sum_{j\neq i}^{N} v_i v_j\right]$$

$$= \frac{1}{n}\left(1-\frac{n}{N}\right)\frac{1}{N(N-1)}\left[(N-1)\sum_{i=1}^{N} v_i^2 - \left(\sum_{i=1}^{N} v_i\right)^2 + \sum_{i=1}^{N} v_i^2\right]$$

$$= \frac{1}{n}\left(1-\frac{n}{N}\right)\frac{1}{N(N-1)}\left[N\sum_{i=1}^{N} v_i^2 - \left(\sum_{i=1}^{N} v_i\right)^2\right]$$

$$= \frac{1}{n}\left(1-\frac{n}{N}\right)\cdot\frac{1}{N-1}\sum_{i=1}^{N}(v_i - \bar{v})^2$$