

Tumor Size Study: Inter-rater reliability with a fixed second factor*

(1.) Data:

Recall the study designed to determine the reliability of size measurements of tumors in cancer patients, where two researchers recruited 26 oncologists to measure simulated tumors. The simulated tumors were made of one of two materials chosen to physically resemble the texture and size of tumors which are found in cancer patients and they were made in one of three shapes: “small,” “oblong,” and “large”. Two copies of each simulated tumor were made; all were placed randomly in rows on a folded blanket and then covered with a sheet of half-inch foam. The oncologists then independently measured each tumor with their usual equipment (ruler and calipers) and recorded the size obtained. “Size” is “cross-sectional area,” which they define as the product of the longest dimension and the shortest dimension of a tumor. Here we will consider shape and oncologist effects on the size measurement.

Questions: Is there large oncologist-to-oncologist variability in measurement of simulated tumor sizes? Does this vary by tumor shape?

(2.) Exploring the data:

Shown on the following pages are simple statistics and plots exploring means, spread, and normality for these data.

```
DATA onc;
INFILE "oncology2.dat" FIRSTOBS=2;
INPUT size oncologist material shape which;
rep = which + 2*(material EQ 2);
PROC SORT DATA=onc OUT=sorted;
BY oncologist shape;
RUN;

%INCLUDE "onc.readdata.sas";

ODS SELECT Moments;
RUN;
PROC UNIVARIATE DATA=sorted;
CLASS shape;
VAR size;
RUN;
```

*File: ~/Teaching/Anova/Twoway/Mixed/onc.mixed.tex; last modified March 19, 2007.

```
=====
The UNIVARIATE Procedure
Variable: size
shape =      1

      Moments

N          104    Sum Weights        104
Mean      1000.56731  Sum Observations   104059
Std Deviation  231.658845 Variance       53665.8207
Skewness     0.25507257 Kurtosis      -0.0105451
Uncorrected SS 109645613  Corrected SS   5527579.53
Coeff Variation 23.1527498 Std Error Mean  22.7160187

=====
The UNIVARIATE Procedure
Variable: size
shape =      2

      Moments

N          104    Sum Weights        104
Mean      1218.11538  Sum Observations   126684
Std Deviation  247.682082 Variance       61346.4137
Skewness     0.51470498 Kurtosis      0.90767221
Uncorrected SS 160634410  Corrected SS   6318680.62
Coeff Variation 20.3332201 Std Error Mean  24.2872263

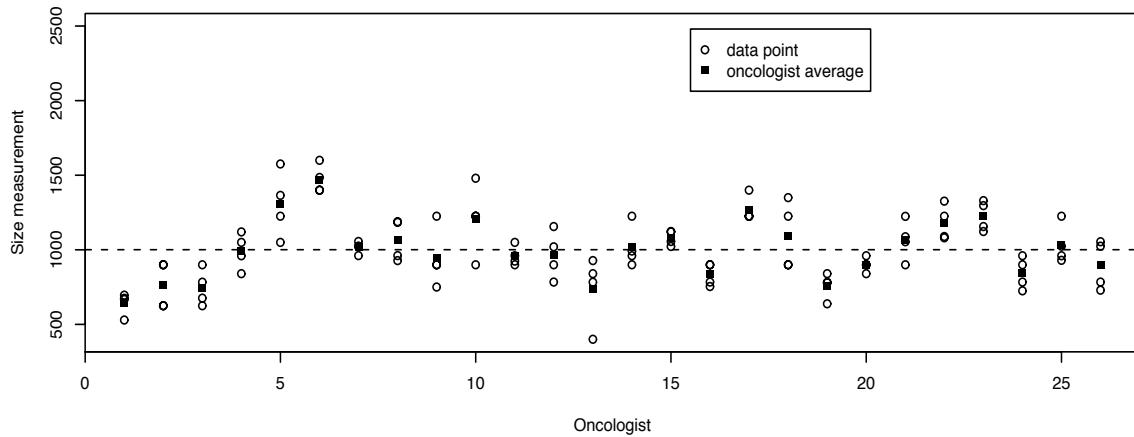
=====
The UNIVARIATE Procedure
Variable: size
shape =      3

      Moments

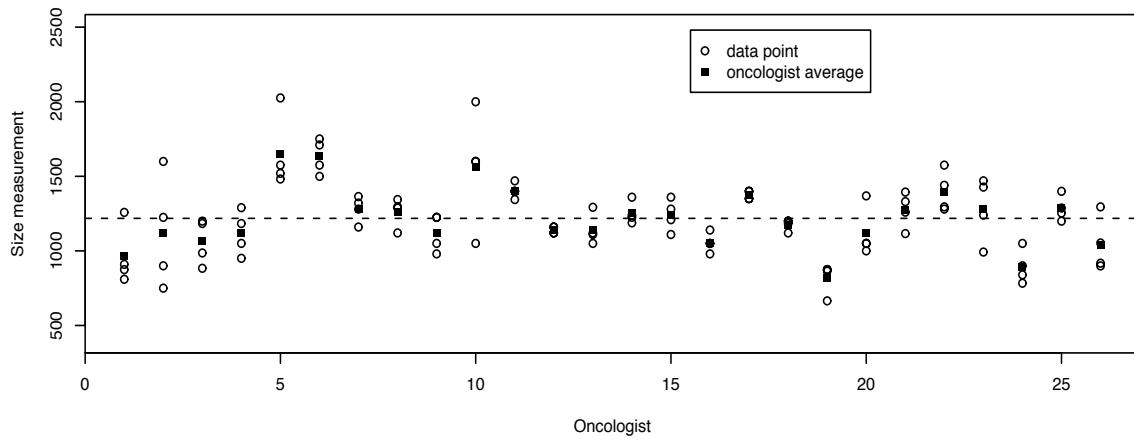
N          104    Sum Weights        104
Mean      1485.54808  Sum Observations   154497
Std Deviation  317.407211 Variance       100747.337
Skewness     1.00747702 Kurtosis      1.30404559
Uncorrected SS 239889697  Corrected SS   10376975.8
Coeff Variation 21.3663372 Std Error Mean  31.1243377
```

Tumor Size Study

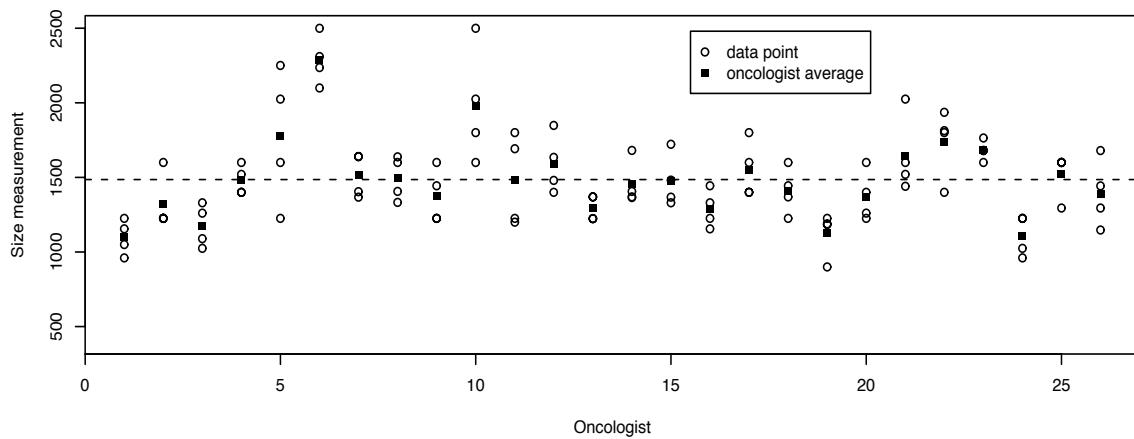
Shape 1 (Small)



Shape 2 (Oblong)

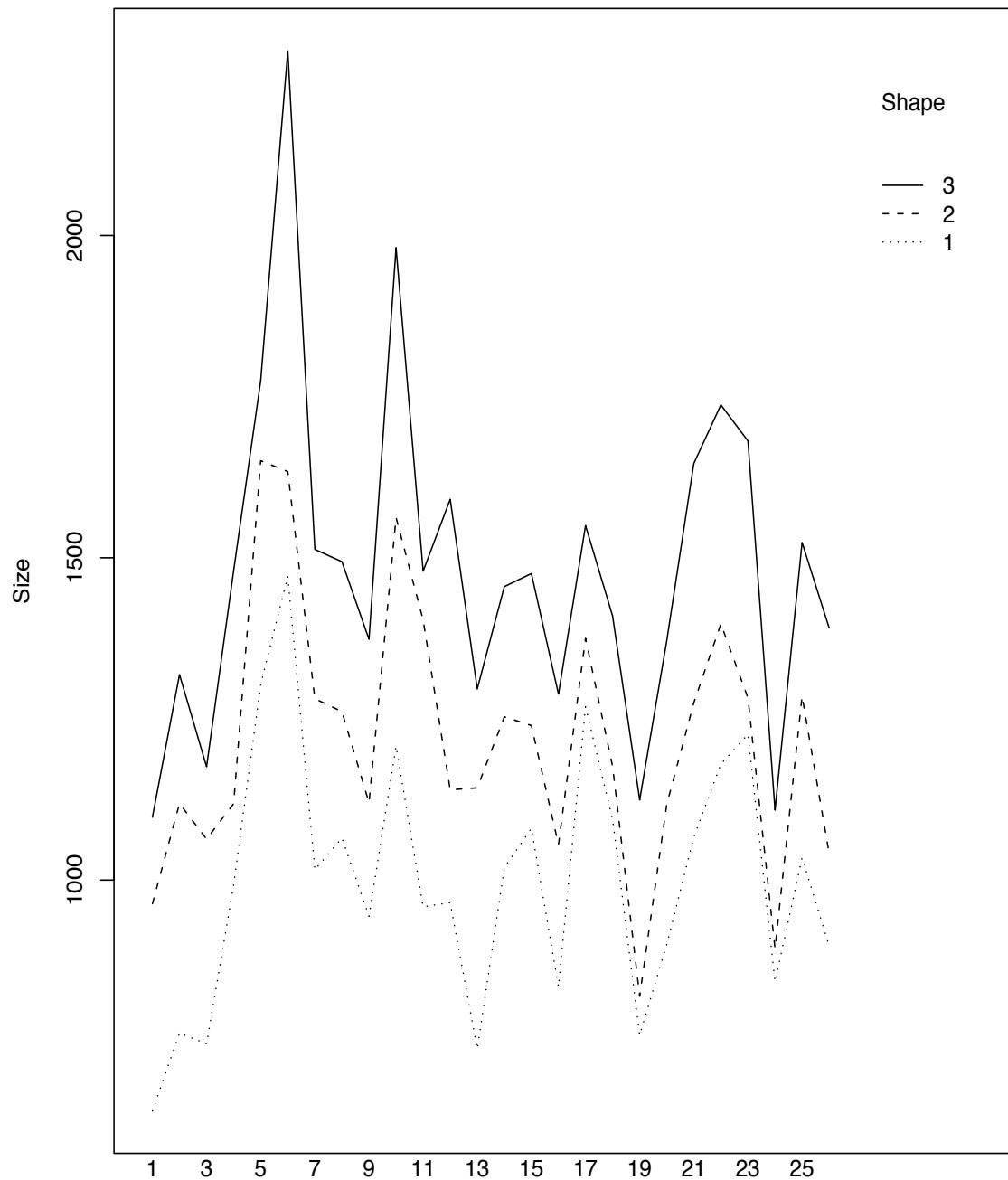


Shape 3 (Large)



onc.twoway.overall.ps

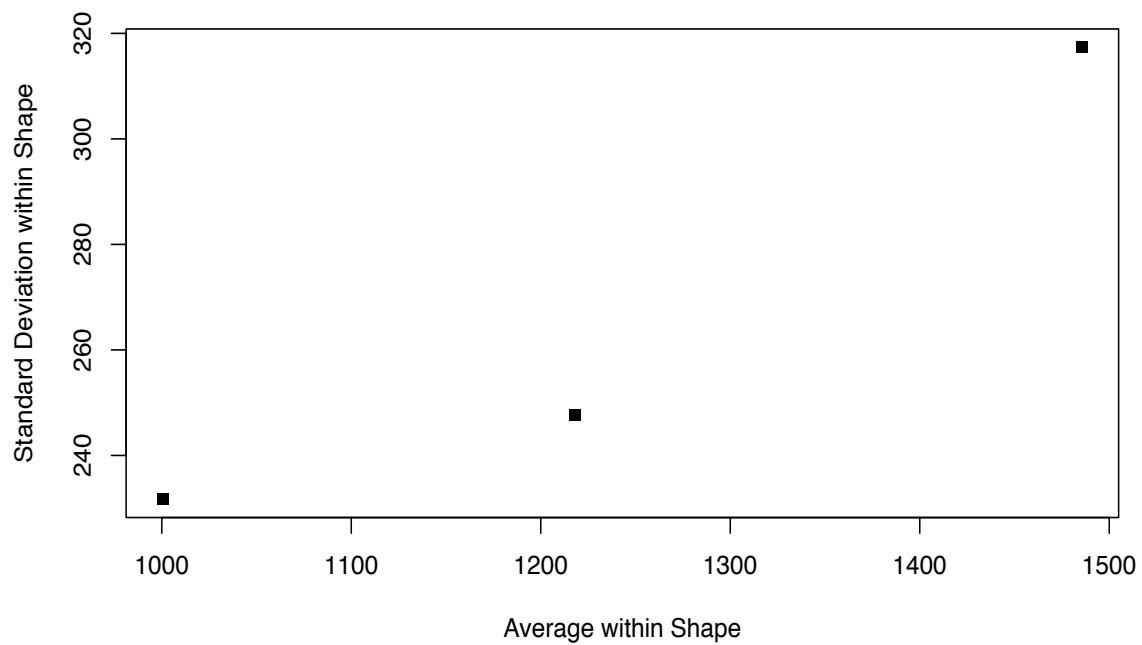
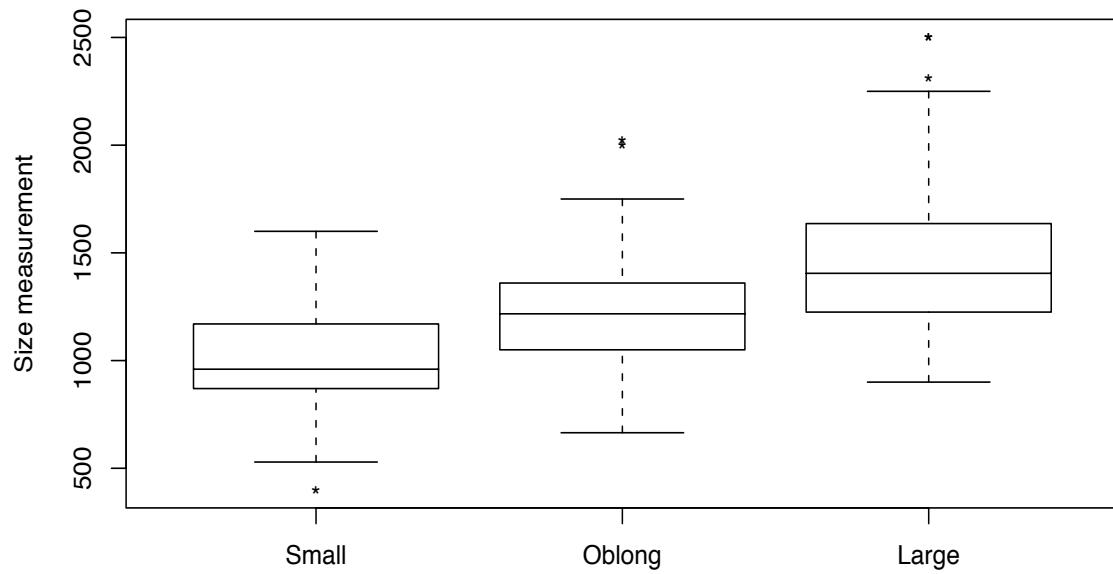
Tumor Size Study



Oncologist

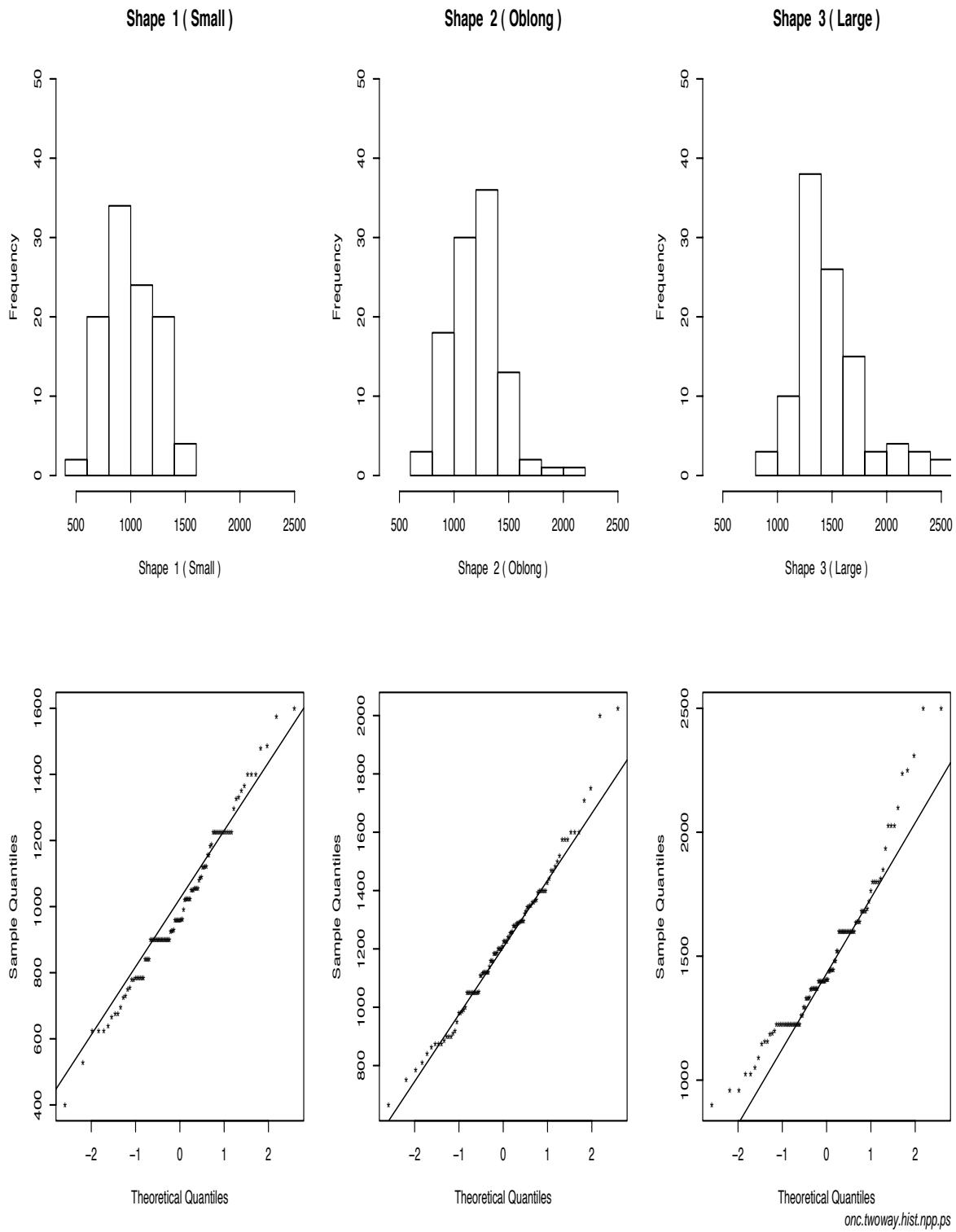
onc.twoway.interact.ps

Tumor Size Study



onc.twoway.boxplot.ps

Tumor Size Study



(3.) Two-way mixed effects model:

Since oncologist is a random effect, anything that interacts with oncologist must be a random effect. We will first test whether the oncologist by shape interaction is needed.

```
%INCLUDE "onc.readdata.sas";  
  
PROC MIXED DATA=sorted METHOD=REML COVTEST;  
  CLASS shape oncologist;  
  MODEL size = shape / SOLUTION;  
  RANDOM INT / SUB=oncologist SOLUTION;  
  RANDOM INT / SUB=shape*oncologist SOLUTION;  
RUN;  
  
=====
```

The SAS System 1
The Mixed Procedure
Model Information
Data Set WORK.SORTED
Dependent Variable size
Covariance Structure Variance Components
Subject Effects oncologist,
shape*oncologist
Estimation Method REML
Residual Variance Method Profile
Fixed Effects SE Method Model-Based
Degrees of Freedom Method Containment

Class Level Information
Class Levels Values
shape 3 1 2 3
oncologist 26 1 2 3 4 5 6 7 8 9 10 11 12 13
14 15 16 17 18 19 20 21 22 23
24 25 26

Dimensions
Covariance Parameters 3
Columns in X 4
Columns in Z Per Subject 4
Subjects 26
Max Obs Per Subject 12

Number of Observations
Number of Observations Read 312
Number of Observations Used 312
Number of Observations Not Used 0

Iteration History
Iteration Evaluations -2 Res Log Like Criterion
0 1 4346.47926086
1 2 4144.21006267 0.00000000

Convergence criteria met.

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z
Intercept	oncologist	43716	13061	3.35	0.0004
Intercept	shape*oncologist	0	.	.	.
Residual		29477	2473.65	11.92	<.0001

Fit Statistics

-2 Res Log Likelihood	4144.2
AIC (smaller is better)	4148.2
AICC (smaller is better)	4148.2
BIC (smaller is better)	4150.7

Solution for Fixed Effects

Effect	shape	Estimate	Standard	DF	t Value	Pr > t
			Error			
Intercept		1485.55	44.3262	25	33.51	<.0001
shape	1	-484.98	23.8089	50	-20.37	<.0001
shape	2	-267.43	23.8089	50	-11.23	<.0001
shape	3	0

Solution for Random Effects

Effect	shape	oncologist	Std Err	Pred	DF	t Value	Pr > t
			Pred				
Intercept		1	-316.15	62.5911	234	-5.05	<.0001
Intercept	1	1	0
Intercept	2	1	0
Intercept	3	1	0
Intercept	2	2	-159.14	62.5911	234	-2.54	0.0117
Intercept	1	2	0
Intercept	2	2	0
Intercept	3	2	0
Intercept	3	3	-226.91	62.5911	234	-3.63	0.0004
Intercept	1	3	0
Intercept	2	3	0
Intercept	3	3	0
Intercept	4	4	-35.7356	62.5911	234	-0.57	0.5686
Intercept	1	4	0
Intercept	2	4	0
Intercept	3	4	0
Intercept	5	5	323.50	62.5911	234	5.17	<.0001
Intercept	1	5	0
Intercept	2	5	0
Intercept	3	5	0
Intercept	6	6	532.50	62.5911	234	8.51	<.0001
Intercept	1	6	0
Intercept	2	6	0
Intercept	3	6	0
Intercept	7	7	33.4596	62.5911	234	0.53	0.5935
Intercept	1	7	0
Intercept	2	7	0
Intercept	3	7	0
Intercept	8	8	36.7734	62.5911	234	0.59	0.5574
Intercept	1	8	0
Intercept	2	8	0
Intercept	3	8	0
Intercept	9	9	-84.2590	62.5911	234	-1.35	0.1795
Intercept	1	9	0
Intercept	2	9	0
Intercept	3	9	0
Intercept	10	10	330.44	62.5911	234	5.28	<.0001
Intercept	1	10	0
Intercept	2	10	0
Intercept	3	10	0
Intercept	11	11	43.2432	62.5911	234	0.69	0.4903
Intercept	1	11	0
Intercept	2	11	0
Intercept	3	11	0
Intercept	12	12	-2.6765	62.5911	234	-0.04	0.9659

Intercept	1	12	0
Intercept	2	12	0
Intercept	3	12	0
Intercept	13	-166.31	62.5911	234	-2.66	0.0084	
Intercept	1	13	0
Intercept	2	13	0
Intercept	3	13	0
Intercept	14	7.4227	62.5911	234	0.12	0.9057	
Intercept	1	14	0
Intercept	2	14	0
Intercept	3	14	0
Intercept	15	28.8045	62.5911	234	0.46	0.6458	
Intercept	1	15	0
Intercept	2	15	0
Intercept	3	15	0
Intercept	16	-166.08	62.5911	234	-2.65	0.0085	
Intercept	1	16	0
Intercept	2	16	0
Intercept	3	16	0
Intercept	17	154.49	62.5911	234	2.47	0.0143	
Intercept	1	17	0
Intercept	2	17	0
Intercept	3	17	0
Intercept	18	-7.8839	62.5911	234	-0.13	0.8999	
Intercept	1	18	0
Intercept	2	18	0
Intercept	3	18	0
Intercept	19	-315.83	62.5911	234	-5.05	<.0001	
Intercept	1	19	0
Intercept	2	19	0
Intercept	3	19	0
Intercept	20	-99.6445	62.5911	234	-1.59	0.1127	
Intercept	1	20	0
Intercept	2	20	0
Intercept	3	20	0
Intercept	21	89.4786	62.5911	234	1.43	0.1542	
Intercept	1	21	0
Intercept	2	21	0
Intercept	3	21	0
Intercept	22	192.68	62.5911	234	3.08	0.0023	
Intercept	1	22	0
Intercept	2	22	0
Intercept	3	22	0
Intercept	23	153.23	62.5911	234	2.45	0.0151	
Intercept	1	23	0
Intercept	2	23	0
Intercept	3	23	0
Intercept	24	-271.41	62.5911	234	-4.34	<.0001	
Intercept	1	24	0
Intercept	2	24	0
Intercept	3	24	0
Intercept	25	44.0322	62.5911	234	0.70	0.4825	
Intercept	1	25	0
Intercept	2	25	0
Intercept	3	25	0
Intercept	26	-118.03	62.5911	234	-1.89	0.0606	
Intercept	1	26	0
Intercept	2	26	0
Intercept	3	26	0

Type 3 Tests of Fixed Effects

Effect	Num DF	Den DF	F Value	Pr > F
shape	2	50	208.19	<.0001

Likelihood ratio tests for models with multiple random effects (intercept random effects like we have here) are not yet established. An approximate Z-test can be carried out. Because it is based on assuming the estimated variance component is normally distributed, the approximation will be

better for larger variance components based on larger sample sizes, but could be quite poor for either smaller variance components or smaller sample sizes.

IMPORTANT NOTE: The log file contains this message: “**NOTE: Estimated G matrix is not positive definite.**” which the listing file does not contain! Always scan your log file. The G matrix is the variance matrix for the random effects. Variance matrices should always be positive definite, so this is telling you that the model is too complex for these data. We need to re-fit it without the interaction.

```
%INCLUDE "onc.readdata.sas";

PROC MIXED DATA=sorted METHOD=REML COVTEST;
  CLASS shape oncologist;
  MODEL size = shape / SOLUTION;
  RANDOM INT / SUB=oncologist SOLUTION;
RUN;

PROC MIXED DATA=sorted METHOD=REML COVTEST;
  CLASS shape oncologist;
  MODEL size = shape / SOLUTION;
RUN;

=====
The Mixed Procedure

      Model Information

Data Set           WORK.SORTED
Dependent Variable   size
Covariance Structure  Variance Components
Subject Effect        oncologist
Estimation Method     REML
Residual Variance Method  Profile
Fixed Effects SE Method  Model-Based
Degrees of Freedom Method Containment

      Class Level Information

Class      Levels    Values
shape          3       1 2 3
oncologist     26      1 2 3 4 5 6 7 8 9 10 11 12 13
                  14 15 16 17 18 19 20 21 22 23
                  24 25 26

Dimensions
Covariance Parameters      2
Columns in X                4
Columns in Z Per Subject    1
Subjects                     26
Max Obs Per Subject         12

Number of Observations
Number of Observations Read 312
Number of Observations Used 312
Number of Observations Not Used 0

      Iteration History

Iteration   Evaluations   -2 Res Log Like      Criterion
          0            1      4346.47926086
          1            1      4144.21006267      0.00000000

      Convergence criteria met.

      Covariance Parameter Estimates

      Standard      Z
Cov Parm   Subject   Estimate   Error   Value   Pr Z
Intercept  oncologist 43716    13061   3.35   0.0004
Residual          29477    2473.64  11.92  <.0001
```

Fit Statistics
-2 Res Log Likelihood 4144.2
AIC (smaller is better) 4148.2
AICC (smaller is better) 4148.2
BIC (smaller is better) 4150.7

Solution for Fixed Effects

Effect	shape	Standard				
		Estimate	Error	DF	t Value	Pr > t
Intercept	1	1485.55	44.3264	25	33.51	<.0001
shape	1	-484.98	23.8089	284	-20.37	<.0001
shape	2	-267.43	23.8089	284	-11.23	<.0001
shape	3	0

Solution for Random Effects

Effect	oncologist	Std Err				
		Estimate	Pred	DF	t Value	Pr > t
Intercept	1	-316.15	62.5913	284	-5.05	<.0001
Intercept	2	-159.14	62.5913	284	-2.54	0.0115
Intercept	3	-226.91	62.5913	284	-3.63	0.0003
Intercept	4	-35.7356	62.5913	284	-0.57	0.5685
Intercept	5	323.50	62.5913	284	5.17	<.0001
Intercept	6	532.50	62.5913	284	8.51	<.0001
Intercept	7	33.4597	62.5913	284	0.53	0.5934
Intercept	8	36.7735	62.5913	284	0.59	0.5573
Intercept	9	-84.2591	62.5913	284	-1.35	0.1793
Intercept	10	330.44	62.5913	284	5.28	<.0001
Intercept	11	43.2432	62.5913	284	0.69	0.4902
Intercept	12	-2.6765	62.5913	284	-0.04	0.9659
Intercept	13	-166.32	62.5913	284	-2.66	0.0083
Intercept	14	7.4227	62.5913	284	0.12	0.9057
Intercept	15	28.8046	62.5913	284	0.46	0.6457
Intercept	16	-166.08	62.5913	284	-2.65	0.0084
Intercept	17	154.49	62.5913	284	2.47	0.0142
Intercept	18	-7.8839	62.5913	284	-0.13	0.8999
Intercept	19	-315.83	62.5913	284	-5.05	<.0001
Intercept	20	-99.6446	62.5913	284	-1.59	0.1125
Intercept	21	89.4786	62.5913	284	1.43	0.1539
Intercept	22	192.68	62.5913	284	3.08	0.0023
Intercept	23	153.23	62.5913	284	2.45	0.0150
Intercept	24	-271.41	62.5913	284	-4.34	<.0001
Intercept	25	44.0322	62.5913	284	0.70	0.4823
Intercept	26	-118.03	62.5913	284	-1.89	0.0604

Type 3 Tests of Fixed Effects

Effect	Num	Den	F Value	Pr > F
	DF	DF		
shape	2	284	208.19	<.0001

=====

The Mixed Procedure

Model Information

Data Set WORK.SORTED
Dependent Variable size
Covariance Structure Diagonal
Estimation Method REML
Residual Variance Method Profile
Fixed Effects SE Method Model-Based
Degrees of Freedom Method Residual

Class Level Information

Class	Levels	Values
shape	3	1 2 3
oncologist	26	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26

Dimensions

Covariance Parameters 1
Columns in X 4

```

Columns in Z          0
Subjects            1
Max Obs Per Subject 312

      Number of Observations
Number of Observations Read      312
Number of Observations Used      312
Number of Observations Not Used  0

      Covariance Parameter Estimates

      Standard      Z
Cov Parm   Estimate    Error   Value   Pr Z
Residual    71920     5786.08  12.43   <.0001

      Fit Statistics

-2 Res Log Likelihood      4346.5
AIC (smaller is better)      4348.5
AICC (smaller is better)     4348.5
BIC (smaller is better)      4352.2

      Solution for Fixed Effects

      Standard
Effect   shape   Estimate    Error   DF   t Value   Pr > |t|
Intercept      1485.55  26.2971  309   56.49   <.0001
shape        1       -484.98  37.1897  309  -13.04   <.0001
shape        2       -267.43  37.1897  309   -7.19   <.0001
shape        3           0       .       .       .       .

      Type 3 Tests of Fixed Effects

      Num   Den
Effect   DF     DF   F Value   Pr >
shape     2     309   85.33   <.0001

```

Does a likelihood ratio test for the oncologist-to-oncologist variability give you a different answer than the Z-test?

Report the results of this study in language a non-statistician could understand.

(4.) Diagnostics:

We examine the same types of plots and statistics for diagnosing mixed models that we did for the one-way and two-way random models. Is there any evidence that model assumptions were violated?

```
%INCLUDE "onc.readdata.sas";

ODS LISTING CLOSE;
RUN;
PROC MIXED DATA=sorted METHOD=REML;
  CLASS shape oncologist;
  MODEL size = shape / OUTP = csfits;
  RANDOM INT / SUB = oncologist;
RUN;
ODS LISTING;
RUN;
DATA csfits;
  SET csfits;
  KEEP size shape oncologist rep Pred Resid;
PROC PRINT DATA=csfits;
  WHERE oncologist LE 2;
  TITLE "PARTIAL LISTING OF DIAGNOSTIC DATA SET csfits";
DATA _NULL_;
  SET csfits;
  FILE "onc.diag.dat";
  IF _N_ = 1 THEN PUT "size shape oncologist rep Pred Resid";
  PUT size shape oncologist rep Pred Resid;
RUN;

ODS LISTING CLOSE;
ODS OUTPUT SolutionR = ranfits;
PROC MIXED DATA=sorted METHOD=REML;
  CLASS shape oncologist;
  MODEL size = shape;
  RANDOM INT / SUB=oncologist SOLUTION;
RUN;
ODS OUTPUT CLOSE;
ODS LISTING;
PROC PRINT DATA=ranfits;
  TITLE "COMPLETE RANDOM EFFECTS DATA SET ranfits";
RUN;
DATA _NULL_;
  SET ranfits;
  FILE "onc.ranfits.dat";
  IF _N_ = 1 THEN PUT "effect oncologist estimate";
  PUT effect oncologist estimate;
RUN;
```

PARTIAL LISTING OF DIAGNOSTIC DATA SET csfits

1

Obs	size	oncologist	shape	rep	Pred	Resid
1	667	1	1	1	684.42	-17.421
2	529	1	1	2	684.42	-155.421
3	696	1	1	3	684.42	11.579
4	675	1	1	4	684.42	-9.421
5	1258	1	2	1	901.97	356.031
6	810	1	2	2	901.97	-91.969
7	875	1	2	3	901.97	-26.969
8	910	1	2	4	901.97	8.031
9	1050	1	3	1	1169.40	-119.402
10	960	1	3	2	1169.40	-209.402
11	1225	1	3	3	1169.40	55.598
12	1155	1	3	4	1169.40	-14.402
13	625	2	1	1	841.43	-216.432
14	900	2	1	2	841.43	58.568
15	625	2	1	3	841.43	-216.432
16	900	2	1	4	841.43	58.568
17	750	2	2	1	1058.98	-308.980
18	1225	2	2	2	1058.98	166.020

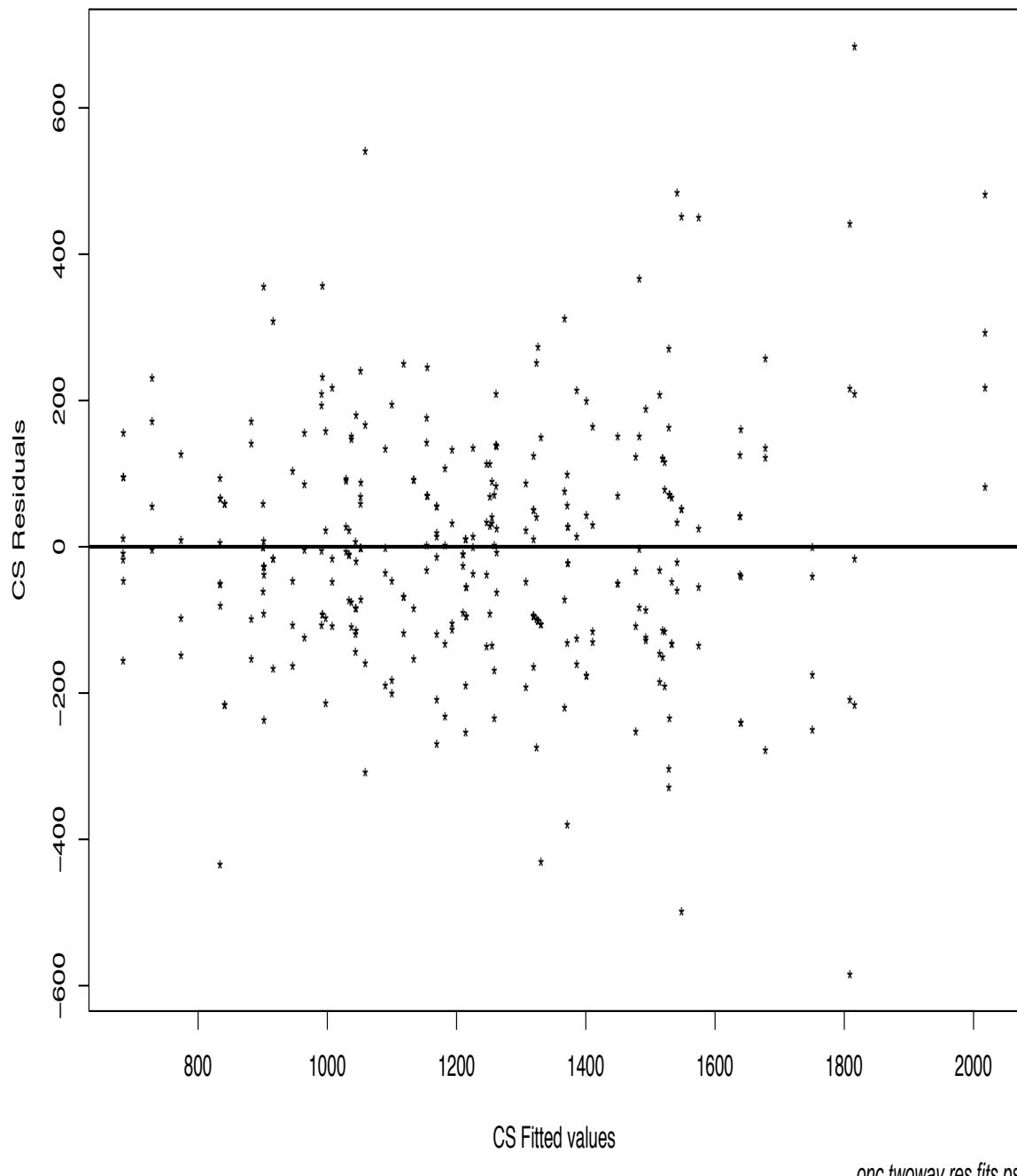
19	900	2	2	3	1058.98	-158.980
20	1600	2	2	4	1058.98	541.020
21	1225	2	3	1	1326.41	-101.413
22	1225	2	3	2	1326.41	-101.413
23	1225	2	3	3	1326.41	-101.413
24	1600	2	3	4	1326.41	273.587

COMPLETE RANDOM EFFECTS DATA SET ranfits

2

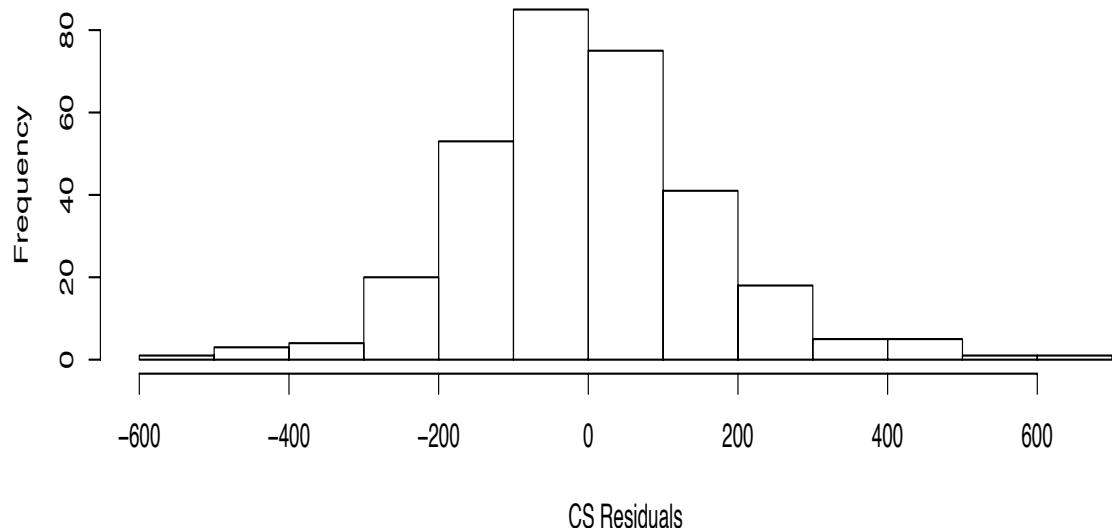
Obs	Effect	oncologist	Estimate	StdErr	Pred	DF	tValue	Probt
1	Intercept	1	-316.15	62.5913	284		-5.05	<.0001
2	Intercept	2	-159.14	62.5913	284		-2.54	0.0115
3	Intercept	3	-226.91	62.5913	284		-3.63	0.0003
4	Intercept	4	-35.7356	62.5913	284		-0.57	0.5685
5	Intercept	5	323.50	62.5913	284		5.17	<.0001
6	Intercept	6	532.50	62.5913	284		8.51	<.0001
7	Intercept	7	33.4597	62.5913	284		0.53	0.5934
8	Intercept	8	36.7735	62.5913	284		0.59	0.5573
9	Intercept	9	-84.2591	62.5913	284		-1.35	0.1793
10	Intercept	10	330.44	62.5913	284		5.28	<.0001
11	Intercept	11	43.2432	62.5913	284		0.69	0.4902
12	Intercept	12	-2.6765	62.5913	284		-0.04	0.9659
13	Intercept	13	-166.32	62.5913	284		-2.66	0.0083
14	Intercept	14	7.4227	62.5913	284		0.12	0.9057
15	Intercept	15	28.8046	62.5913	284		0.46	0.6457
16	Intercept	16	-166.08	62.5913	284		-2.65	0.0084
17	Intercept	17	154.49	62.5913	284		2.47	0.0142
18	Intercept	18	-7.8839	62.5913	284		-0.13	0.8999
19	Intercept	19	-315.83	62.5913	284		-5.05	<.0001
20	Intercept	20	-99.6446	62.5913	284		-1.59	0.1125
21	Intercept	21	89.4786	62.5913	284		1.43	0.1539
22	Intercept	22	192.68	62.5913	284		3.08	0.0023
23	Intercept	23	153.23	62.5913	284		2.45	0.0150
24	Intercept	24	-271.41	62.5913	284		-4.34	<.0001
25	Intercept	25	44.0322	62.5913	284		0.70	0.4823
26	Intercept	26	-118.03	62.5913	284		-1.89	0.0604

Tumor Size Study
CS residuals vs. CS fitted values

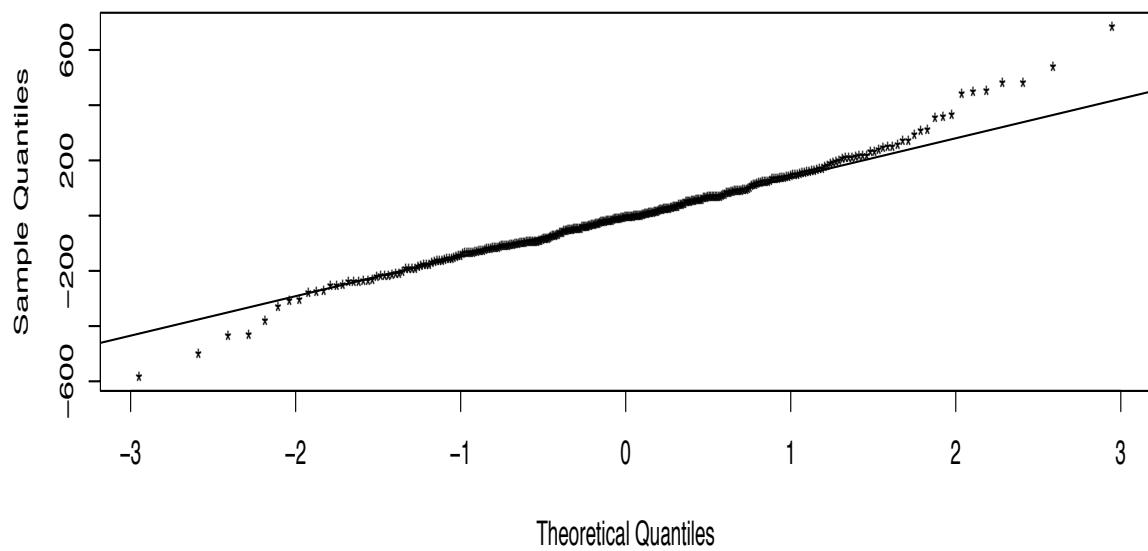


onc.twoway.res.fits.ps

Tumor Size Study
Histograms of CS residuals



CS Residuals

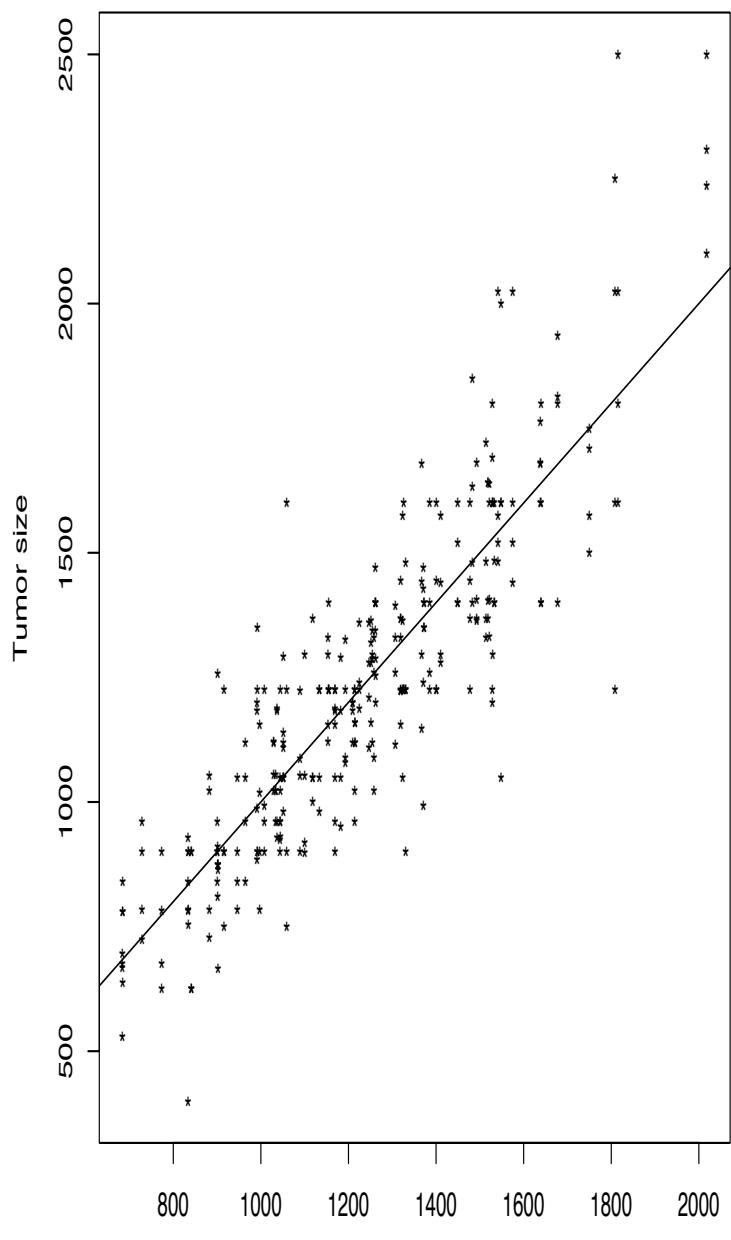


Theoretical Quantiles

onc.twoway.res.hist.npp.ps

Tumor Size Study

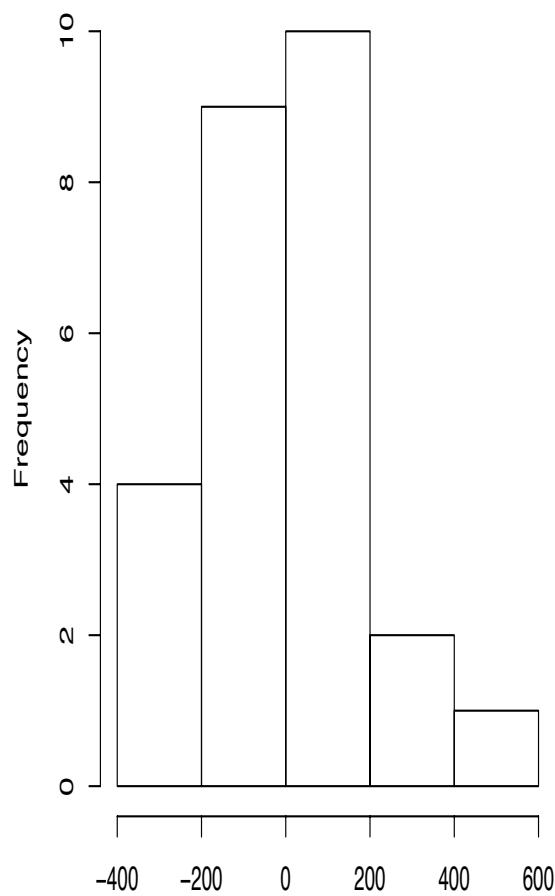
Observed vs. CS fitted values



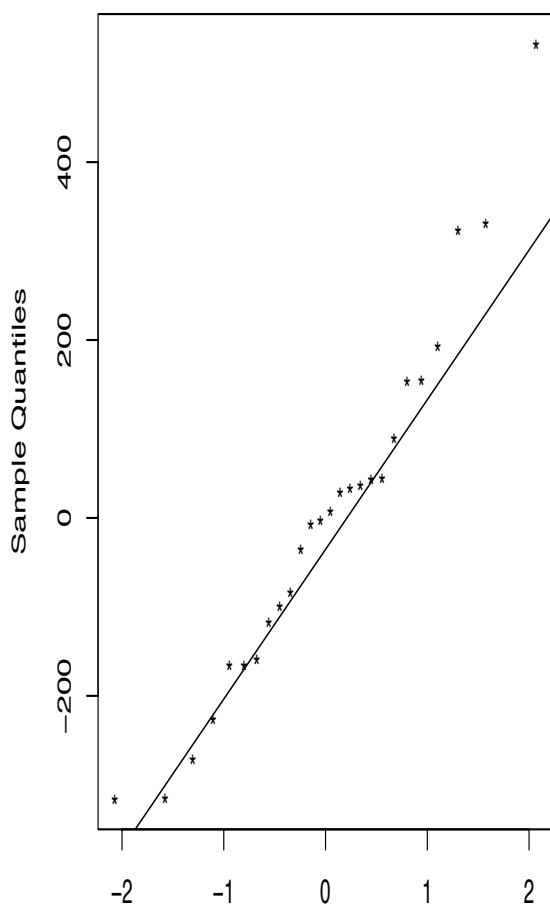
onc.twoway.obs.fits.ps

Tumor Size Study

Histograms and quantile plots of estimated random effects



Estimated random oncologist effects



Theoretical Quantiles

onc.twoway.ranfits.ps

(5.) Fitting the model in PROC GLM:

Compare the results below to those from PROC MIXED with the interaction. Are the estimated variance components different? Is the test of the shape main effect different? Are the estimated random effects different?

```
%INCLUDE "onc.readdata.sas";

PROC GLM DATA=sorted;
  CLASS shape oncologist;
  MODEL size = shape oncologist shape*oncologist / SOLUTION;
  RANDOM oncologist shape*oncologist / TEST;
RUN;

=====
1

The SAS System
The GLM Procedure

      Class Level Information

Class      Levels      Values
shape          3      1 2 3
oncologist     26      1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26

Number of Observations Read      312
Number of Observations Used    312

=====
2

The SAS System
The GLM Procedure

Dependent Variable: size

      Sum of
Source      DF      Squares      Mean Square      F Value      Pr > F
Model       77      27575930.49      358128.97      12.11      <.0001
Error      234      6921169.00      29577.65
Corrected Total      311      34497099.49

      R-Square      Coeff Var      Root MSE      size Mean
0.799370      13.92852      171.9815      1234.744

      Source      DF      Type I SS      Mean Square      F Value      Pr > F
shape          2      12273863.58      6136931.79      207.49      <.0001
oncologist     25      13851803.32      554072.13      18.73      <.0001
shape*oncologist  50      1450263.58      29005.27      0.98      0.5162

      Source      DF      Type III SS      Mean Square      F Value      Pr > F
shape          2      12273863.58      6136931.79      207.49      <.0001
oncologist     25      13851803.32      554072.13      18.73      <.0001
shape*oncologist  50      1450263.58      29005.27      0.98      0.5162

      Standard
Parameter      Estimate      Error      t Value      Pr > |t|
```

Intercept		1391.250000 B	85.9907630	16.18	<.0001
shape	1	-493.500000 B	121.6093033	-4.06	<.0001
shape	2	-350.000000 B	121.6093033	-2.88	0.0044
shape	3	0.000000 B	.	.	.
oncologist	1	-293.750000 B	121.6093033	-2.42	0.0165
oncologist	2	-72.500000 B	121.6093033	-0.60	0.5516
oncologist	3	-215.500000 B	121.6093033	-1.77	0.0777
oncologist	4	88.750000 B	121.6093033	0.73	0.4662
oncologist	5	383.750000 B	121.6093033	3.16	0.0018
oncologist	6	895.250000 B	121.6093033	7.36	<.0001
oncologist	7	121.750000 B	121.6093033	1.00	0.3178
oncologist	8	102.750000 B	121.6093033	0.84	0.3990
oncologist	9	-17.750000 B	121.6093033	-0.15	0.8841
oncologist	10	590.000000 B	121.6093033	4.85	<.0001
oncologist	11	88.000000 B	121.6093033	0.72	0.4700
oncologist	12	199.500000 B	121.6093033	1.64	0.1022
oncologist	13	-94.750000 B	121.6093033	-0.78	0.4367
oncologist	14	64.000000 B	121.6093033	0.53	0.5992
oncologist	15	84.250000 B	121.6093033	0.69	0.4891
oncologist	16	-102.750000 B	121.6093033	-0.84	0.3990
oncologist	17	158.750000 B	121.6093033	1.31	0.1930
oncologist	18	18.250000 B	121.6093033	0.15	0.8808
oncologist	19	-267.000000 B	121.6093033	-2.20	0.0291
oncologist	20	-20.000000 B	121.6093033	-0.16	0.8695
oncologist	21	255.000000 B	121.6093033	2.10	0.0371
oncologist	22	346.000000 B	121.6093033	2.85	0.0048
oncologist	23	290.000000 B	121.6093033	2.38	0.0179
oncologist	24	-282.750000 B	121.6093033	-2.33	0.0209
oncologist	25	132.500000 B	121.6093033	1.09	0.2770
oncologist	26	0.000000 B	.	.	.
shape*oncologist	1 1	37.750000 B	171.9815260	0.22	0.8265
shape*oncologist	1 2	-62.750000 B	171.9815260	-0.36	0.7155
shape*oncologist	1 3	63.750000 B	171.9815260	0.37	0.7112
shape*oncologist	1 4	6.000000 B	171.9815260	0.03	0.9722
shape*oncologist	1 5	22.250000 B	171.9815260	0.13	0.8972
shape*oncologist	1 6	-321.750000 B	171.9815260	-1.87	0.0626
shape*oncologist	1 7	-3.500000 B	171.9815260	-0.02	0.9838
shape*oncologist	1 8	64.750000 B	171.9815260	0.38	0.7069
shape*oncologist	1 9	63.750000 B	171.9815260	0.37	0.7112
shape*oncologist	1 10	-280.250000 B	171.9815260	-1.63	0.1045
shape*oncologist	1 11	-27.250000 B	171.9815260	-0.16	0.8742
shape*oncologist	1 12	-132.250000 B	171.9815260	-0.77	0.4427
shape*oncologist	1 13	-65.250000 B	171.9815260	-0.38	0.7047
shape*oncologist	1 14	57.500000 B	171.9815260	0.33	0.7384
shape*oncologist	1 15	98.250000 B	171.9815260	0.57	0.5684
shape*oncologist	1 16	39.500000 B	171.9815260	0.23	0.8185
shape*oncologist	1 17	212.250000 B	171.9815260	1.23	0.2184
shape*oncologist	1 18	177.750000 B	171.9815260	1.03	0.3024
shape*oncologist	1 19	128.750000 B	171.9815260	0.75	0.4548
shape*oncologist	1 20	22.250000 B	171.9815260	0.13	0.8972
shape*oncologist	1 21	-86.250000 B	171.9815260	-0.50	0.6165
shape*oncologist	1 22	-63.750000 B	171.9815260	-0.37	0.7112
shape*oncologist	1 23	38.250000 B	171.9815260	0.22	0.8242
shape*oncologist	1 24	227.250000 B	171.9815260	1.32	0.1877
shape*oncologist	1 25	4.500000 B	171.9815260	0.03	0.9791
shape*oncologist	1 26	0.000000 B	.	.	.
shape*oncologist	2 1	215.750000 B	171.9815260	1.25	0.2109
shape*oncologist	2 2	150.000000 B	171.9815260	0.87	0.3840
shape*oncologist	2 3	237.750000 B	171.9815260	1.38	0.1682
shape*oncologist	2 4	-11.500000 B	171.9815260	-0.07	0.9467
shape*oncologist	2 5	225.500000 B	171.9815260	1.31	0.1911
shape*oncologist	2 6	-302.750000 B	171.9815260	-1.76	0.0797
shape*oncologist	2 7	118.250000 B	171.9815260	0.69	0.4924
shape*oncologist	2 8	117.500000 B	171.9815260	0.68	0.4951
shape*oncologist	2 9	96.500000 B	171.9815260	0.56	0.5753
shape*oncologist	2 10	-68.750000 B	171.9815260	-0.40	0.6897
shape*oncologist	2 11	274.250000 B	171.9815260	1.59	0.1121
shape*oncologist	2 12	-100.750000 B	171.9815260	-0.59	0.5586
shape*oncologist	2 13	196.500000 B	171.9815260	1.14	0.2544
shape*oncologist	2 14	148.000000 B	171.9815260	0.86	0.3904
shape*oncologist	2 15	114.250000 B	171.9815260	0.66	0.5071
shape*oncologist	2 16	116.500000 B	171.9815260	0.68	0.4988
shape*oncologist	2 17	175.000000 B	171.9815260	1.02	0.3099
shape*oncologist	2 18	116.500000 B	171.9815260	0.68	0.4988
shape*oncologist	2 19	45.500000 B	171.9815260	0.26	0.7916
shape*oncologist	2 20	96.000000 B	171.9815260	0.56	0.5772

shape*oncologist 2 21	-21.250000	B	171.9815260	-0.12	0.9018
shape*oncologist 2 22	10.250000	B	171.9815260	0.06	0.9525
shape*oncologist 2 23	-48.750000	B	171.9815260	-0.28	0.7771
shape*oncologist 2 24	135.000000	B	171.9815260	0.78	0.4333
shape*oncologist 2 25	111.500000	B	171.9815260	0.65	0.5174
shape*oncologist 2 26	0.000000	B	.	.	.
shape*oncologist 3 1	0.000000	B	.	.	.
shape*oncologist 3 2	0.000000	B	.	.	.
shape*oncologist 3 3	0.000000	B	.	.	.
shape*oncologist 3 4	0.000000	B	.	.	.
shape*oncologist 3 5	0.000000	B	.	.	.
shape*oncologist 3 6	0.000000	B	.	.	.
shape*oncologist 3 7	0.000000	B	.	.	.
shape*oncologist 3 8	0.000000	B	.	.	.
shape*oncologist 3 9	0.000000	B	.	.	.
shape*oncologist 3 10	0.000000	B	.	.	.
shape*oncologist 3 11	0.000000	B	.	.	.
shape*oncologist 3 12	0.000000	B	.	.	.
shape*oncologist 3 13	0.000000	B	.	.	.
shape*oncologist 3 14	0.000000	B	.	.	.
shape*oncologist 3 15	0.000000	B	.	.	.
shape*oncologist 3 16	0.000000	B	.	.	.
shape*oncologist 3 17	0.000000	B	.	.	.
shape*oncologist 3 18	0.000000	B	.	.	.
shape*oncologist 3 19	0.000000	B	.	.	.
shape*oncologist 3 20	0.000000	B	.	.	.
shape*oncologist 3 21	0.000000	B	.	.	.
shape*oncologist 3 22	0.000000	B	.	.	.
shape*oncologist 3 23	0.000000	B	.	.	.
shape*oncologist 3 24	0.000000	B	.	.	.
shape*oncologist 3 25	0.000000	B	.	.	.
shape*oncologist 3 26	0.000000	B	.	.	.

NOTE: The X'X matrix has been found to be singular, and a generalized inverse was used to solve the normal equations. Terms whose estimates are followed by the letter 'B' are not uniquely estimable.

The SAS System

3

The GLM Procedure

Source	Type III Expected Mean Square
shape	Var(Error) + 4 Var(shape*oncologist) + Q(shape)
oncologist	Var(Error) + 4 Var(shape*oncologist) + 12 Var(oncologist)
shape*oncologist	Var(Error) + 4 Var(shape*oncologist)

The SAS System

4

The GLM Procedure

Tests of Hypotheses for Mixed Model Analysis of Variance

Dependent Variable: size

Source	DF	Type III SS	Mean Square	F Value	Pr > F
shape	2	12273864	6136932	211.58	<.0001
oncologist	25	13851803	554072	19.10	<.0001
Error	50	1450264	29005		
Error: MS(shape*oncologist)					

Source	DF	Type III SS	Mean Square	F Value	Pr > F
shape*oncologist	50	1450264	29005	0.98	0.5162
Error: MS(Error)	234	6921169	29578		

(6.) Comparisons to a two-way fixed effects ANOVA:

How do the PROC GLM results shown below, with both shape and oncologist taken to be fixed factors, compare to the results when oncologist was treated as random? Have the sums of squares or mean squares changed? How have the expected mean squares changed? Have the estimates of each component of variance changed?

```
%INCLUDE "onc.readdata.sas";

PROC GLM DATA=sorted;
  CLASS shape oncologist;
  MODEL size = shape oncologist shape*oncologist / SOLUTION;
RUN;

=====
1

The SAS System
The GLM Procedure

      Class Level Information

Class      Levels      Values
shape          3      1 2 3
oncologist     26      1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26

Number of Observations Read      312
Number of Observations Used    312

=====
2

The SAS System
The GLM Procedure

Dependent Variable: size

      Sum of
Source      DF      Squares      Mean Square      F Value      Pr > F
Model       77      27575930.49      358128.97      12.11      <.0001
Error      234      6921169.00      29577.65
Corrected Total      311      34497099.49

      R-Square      Coeff Var      Root MSE      size Mean
0.799370      13.92852      171.9815      1234.744

      Source      DF      Type I SS      Mean Square      F Value      Pr > F
shape          2      12273863.58      6136931.79      207.49      <.0001
oncologist     25      13851803.32      554072.13      18.73      <.0001
shape*oncologist      50      1450263.58      29005.27      0.98      0.5162

      Source      DF      Type III SS      Mean Square      F Value      Pr > F
shape          2      12273863.58      6136931.79      207.49      <.0001
oncologist     25      13851803.32      554072.13      18.73      <.0001
shape*oncologist      50      1450263.58      29005.27      0.98      0.5162

      Standard
```

Parameter		Estimate	Error	t Value	Pr > t
Intercept		1391.250000 B	85.9907630	16.18	<.0001
shape	1	-493.500000 B	121.6093033	-4.06	<.0001
shape	2	-350.000000 B	121.6093033	-2.88	0.0044
shape	3	0.000000 B	.	.	.
oncologist	1	-293.750000 B	121.6093033	-2.42	0.0165
oncologist	2	-72.500000 B	121.6093033	-0.60	0.5516
oncologist	3	-215.500000 B	121.6093033	-1.77	0.0777
oncologist	4	88.750000 B	121.6093033	0.73	0.4662
oncologist	5	383.750000 B	121.6093033	3.16	0.0018
oncologist	6	895.250000 B	121.6093033	7.36	<.0001
oncologist	7	121.750000 B	121.6093033	1.00	0.3178
oncologist	8	102.750000 B	121.6093033	0.84	0.3990
oncologist	9	-17.750000 B	121.6093033	-0.15	0.8841
oncologist	10	590.000000 B	121.6093033	4.85	<.0001
oncologist	11	88.000000 B	121.6093033	0.72	0.4700
oncologist	12	199.500000 B	121.6093033	1.64	0.1022
oncologist	13	-94.750000 B	121.6093033	-0.78	0.4367
oncologist	14	64.000000 B	121.6093033	0.53	0.5992
oncologist	15	84.250000 B	121.6093033	0.69	0.4891
oncologist	16	-102.750000 B	121.6093033	-0.84	0.3990
oncologist	17	158.750000 B	121.6093033	1.31	0.1930
oncologist	18	18.250000 B	121.6093033	0.15	0.8808
oncologist	19	-267.000000 B	121.6093033	-2.20	0.0291
oncologist	20	-20.000000 B	121.6093033	-0.16	0.8695
oncologist	21	255.000000 B	121.6093033	2.10	0.0371
oncologist	22	346.000000 B	121.6093033	2.85	0.0048
oncologist	23	290.000000 B	121.6093033	2.38	0.0179
oncologist	24	-282.750000 B	121.6093033	-2.33	0.0209
oncologist	25	132.500000 B	121.6093033	1.09	0.2770
oncologist	26	0.000000 B	.	.	.
shape*oncologist	1 1	37.750000 B	171.9815260	0.22	0.8265
shape*oncologist	1 2	-62.750000 B	171.9815260	-0.36	0.7155
shape*oncologist	1 3	63.750000 B	171.9815260	0.37	0.7112
shape*oncologist	1 4	6.000000 B	171.9815260	0.03	0.9722
shape*oncologist	1 5	22.250000 B	171.9815260	0.13	0.8972
shape*oncologist	1 6	-321.750000 B	171.9815260	-1.87	0.0626
shape*oncologist	1 7	-3.500000 B	171.9815260	-0.02	0.9838
shape*oncologist	1 8	64.750000 B	171.9815260	0.38	0.7069
shape*oncologist	1 9	63.750000 B	171.9815260	0.37	0.7112
shape*oncologist	1 10	-280.250000 B	171.9815260	-1.63	0.1045
shape*oncologist	1 11	-27.250000 B	171.9815260	-0.16	0.8742
shape*oncologist	1 12	-132.250000 B	171.9815260	-0.77	0.4427
shape*oncologist	1 13	-65.250000 B	171.9815260	-0.38	0.7047
shape*oncologist	1 14	57.500000 B	171.9815260	0.33	0.7384
shape*oncologist	1 15	98.250000 B	171.9815260	0.57	0.5684
shape*oncologist	1 16	39.500000 B	171.9815260	0.23	0.8185
shape*oncologist	1 17	212.250000 B	171.9815260	1.23	0.2184
shape*oncologist	1 18	177.750000 B	171.9815260	1.03	0.3024
shape*oncologist	1 19	128.750000 B	171.9815260	0.75	0.4548
shape*oncologist	1 20	22.250000 B	171.9815260	0.13	0.8972
shape*oncologist	1 21	-86.250000 B	171.9815260	-0.50	0.6165
shape*oncologist	1 22	-63.750000 B	171.9815260	-0.37	0.7112
shape*oncologist	1 23	38.250000 B	171.9815260	0.22	0.8242
shape*oncologist	1 24	227.250000 B	171.9815260	1.32	0.1877
shape*oncologist	1 25	4.500000 B	171.9815260	0.03	0.9791
shape*oncologist	1 26	0.000000 B	.	.	.
shape*oncologist	2 1	215.750000 B	171.9815260	1.25	0.2109
shape*oncologist	2 2	150.000000 B	171.9815260	0.87	0.3840
shape*oncologist	2 3	237.750000 B	171.9815260	1.38	0.1682
shape*oncologist	2 4	-11.500000 B	171.9815260	-0.07	0.9467
shape*oncologist	2 5	225.500000 B	171.9815260	1.31	0.1911
shape*oncologist	2 6	-302.750000 B	171.9815260	-1.76	0.0797
shape*oncologist	2 7	118.250000 B	171.9815260	0.69	0.4924
shape*oncologist	2 8	117.500000 B	171.9815260	0.68	0.4951
shape*oncologist	2 9	96.500000 B	171.9815260	0.56	0.5753
shape*oncologist	2 10	-68.750000 B	171.9815260	-0.40	0.6897
shape*oncologist	2 11	274.250000 B	171.9815260	1.59	0.1121
shape*oncologist	2 12	-100.750000 B	171.9815260	-0.59	0.5586
shape*oncologist	2 13	196.500000 B	171.9815260	1.14	0.2544
shape*oncologist	2 14	148.000000 B	171.9815260	0.86	0.3904
shape*oncologist	2 15	114.250000 B	171.9815260	0.66	0.5071
shape*oncologist	2 16	116.500000 B	171.9815260	0.68	0.4988
shape*oncologist	2 17	175.000000 B	171.9815260	1.02	0.3099
shape*oncologist	2 18	116.500000 B	171.9815260	0.68	0.4988
shape*oncologist	2 19	45.500000 B	171.9815260	0.26	0.7916

shape*oncologist 2 20	96.000000	B	171.9815260	0.56	0.5772
shape*oncologist 2 21	-21.250000	B	171.9815260	-0.12	0.9018
shape*oncologist 2 22	10.250000	B	171.9815260	0.06	0.9525
shape*oncologist 2 23	-48.750000	B	171.9815260	-0.28	0.7771
shape*oncologist 2 24	135.000000	B	171.9815260	0.78	0.4333
shape*oncologist 2 25	111.500000	B	171.9815260	0.65	0.5174
shape*oncologist 2 26	0.000000	B	.	.	.
shape*oncologist 3 1	0.000000	B	.	.	.
shape*oncologist 3 2	0.000000	B	.	.	.
shape*oncologist 3 3	0.000000	B	.	.	.
shape*oncologist 3 4	0.000000	B	.	.	.
shape*oncologist 3 5	0.000000	B	.	.	.
shape*oncologist 3 6	0.000000	B	.	.	.
shape*oncologist 3 7	0.000000	B	.	.	.
shape*oncologist 3 8	0.000000	B	.	.	.
shape*oncologist 3 9	0.000000	B	.	.	.
shape*oncologist 3 10	0.000000	B	.	.	.
shape*oncologist 3 11	0.000000	B	.	.	.
shape*oncologist 3 12	0.000000	B	.	.	.
shape*oncologist 3 13	0.000000	B	.	.	.
shape*oncologist 3 14	0.000000	B	.	.	.
shape*oncologist 3 15	0.000000	B	.	.	.
shape*oncologist 3 16	0.000000	B	.	.	.
shape*oncologist 3 17	0.000000	B	.	.	.
shape*oncologist 3 18	0.000000	B	.	.	.
shape*oncologist 3 19	0.000000	B	.	.	.
shape*oncologist 3 20	0.000000	B	.	.	.
shape*oncologist 3 21	0.000000	B	.	.	.
shape*oncologist 3 22	0.000000	B	.	.	.
shape*oncologist 3 23	0.000000	B	.	.	.
shape*oncologist 3 24	0.000000	B	.	.	.
shape*oncologist 3 25	0.000000	B	.	.	.
shape*oncologist 3 26	0.000000	B	.	.	.

NOTE: The $X'X$ matrix has been found to be singular, and a generalized inverse was used to solve the normal equations. Terms whose estimates are followed by the letter 'B' are not uniquely estimable.