MATH 1512 — R E V I E W   E X A M   1

How to use this review: Go through the topics listed. You can expect that a problem on any of these topics be on the exam. There are good homework problems on each topic, and a list of sample problems is added below (you should recognize many of them from the homework). Do a sample problem per topic. Do more if you feel shaky at it.

TOPICS COVERED

(§§2.1-2.5, some review material)

1. Review Material

Graph basic functions and translations and transformations

2. Limits

Find limits $\lim_{x\to c} f(x)$, given graph of f.
Find limits $\lim_{x\to c} f(x)$, given expression for f.

3. Continuity

State the definition of continuity at a point. Use it to determine where a function is continuous.

4. The Derivative

State the definition of the derivative as a limit. Use it to find derivatives.
Recognize when a limit is a derivative.
Graphs of functions and their derivatives.
Use power, product, quotient, chain rules to find derivatives.
Give examples of functions for which the derivative does not exist at some point $x = a$.

5. Applications

Find equations of lines tangent or normal to curves $y = f(x)$ at a point.
Find points on graph with zero slope.

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1. Review Material

1. Sketch graphs of
   (a) $y = \sin(3x)$, $f(\theta) = 1 - \cos(2\theta)$,
   (b) $f(x) = |x|$, $f(x) = |x - 1|$,
   (c) $f(x) = 1/x$, $f(x) = 1/(x + 1)$, $f(x) = x + 1/x$
   (d) $f(x) = x^{1/2}$, $f(x) = 1^{1/3}$
   (e) $f(x) = 2\tan x$,
   (f) $f(x) = x^2 - 2x$, $f(x) = x^2 - x^3$
   (g) functions defined piecewise

2. Limits

2. Evaluate the following limits or explain why they do not exist. Show all your work, or explain what you did.

   (a) $\lim_{x\to 0} \dfrac{x^2}{1-x}$ (b) $\lim_{x\to 1} \dfrac{x^2}{1-x}$ (c) $\lim_{x\to 1} \dfrac{1-x^2}{1-x}$

   (d) $\lim_{x\to 2} \dfrac{3x - 5\cos x}{x^2}$ (e) $\lim_{x\to 0} \dfrac{3x - 5\cos x}{x^2}$ (f) $\lim_{x\to -5} \dfrac{x^2 - 25}{x + 5}$

(g,h) $\lim_{x\to 0} f(x)$ and $\lim_{x\to\pi} f(x)$ where $f(x) = \begin{cases} 1+\sin x\ , & \text{if } x<0 \\ \cos x\ , & \text{if } 0\le x\le\pi \\ \sin x\ , & \text{if } x>\pi \end{cases}$

3. Homework problems on Squeeze Theorem: HW 2, 1.6:38,41

3. Continuity

4. Homework problems Day 4: 1.8: 23,42,44,46,47

4. The Derivative

5. State the definition of the derivative of a function $f(x)$ as a limit. Include a graph that illustrates the geometric interpretation of this limit.

6. For the following functions $f(x)$, find $f'(x)$ using the definition of the derivative as a limit. Confirm your result using the rules for differentiation.
   (a) $f(x) = x/(x+2)$ (b) $f(x) = \sqrt{2-x}$ (c) $f(x) = \cos(x)$ (d) $f(x) = x^4$

7. Find the derivatives of the following functions. Simplify your answer.
   (a) $f(x) = (3x+2)^8(x^2+3)^6$ (b) $H(t) = \sqrt[3]{3t}(t+2) + \dfrac{1}{t^2\sqrt{t}}$ (c) $f(x) = \dfrac{x^2+4x+3}{\sqrt{2x}}$
   (d) $y = \dfrac{r}{\sqrt{r^2+1}}$ (e) $v(x) = \sin(x^3-3x)$ (f) $f(x) = (2x-5)\tan(x)$
   (g) $g(t) = \cos(\sqrt{t})$ (h) $g(t) = \sqrt{\cos t}$ (i) $g(t) = \sqrt{t}\cos t$
   (j) $f(u) = \dfrac{1-u^2}{1+u^2}$

8. The following limits all represent the slope of the line tangent to the graph of $y = f(x)$ at some point $x = a$, for some function f. In each case, state f and a. Evaluate the limits, explaining how you obtained the answer.
   (a) $\lim_{h\to 0} \dfrac{\sqrt[4]{16+h}-2}{h}$ (b) $\lim_{x\to 1} \dfrac{x^{17}-1}{x-1}$ (c) $\lim_{x\to 0} \dfrac{\sin(x)}{x}$ (d) $\lim_{x\to 0} \dfrac{\cos(x)-1}{x}$

9. For each of the following functions: (i) sketch a graph of the function, (ii) use your sketch to sketch a graph of its derivative, (iii) find a formula for its derivative and check that the graph is consistent with your results in (ii)
   (a) $f(x) = x^{1/3}$ (b) $f(x) = \sec(x)$
   (c) $f(x) = |x|x$ (d) $f(x) = \sqrt{c^2-x^2}$, $c > 0$.

10. The gravitational force exerted by the earth on a unit mass at a distance r from the center of the planet is

$$F(r) = \begin{cases} \dfrac{GMr}{R^3} & \text{if } 0\le r<R \\ \dfrac{GM}{r^2} & \text{if } r\ge R \end{cases}$$

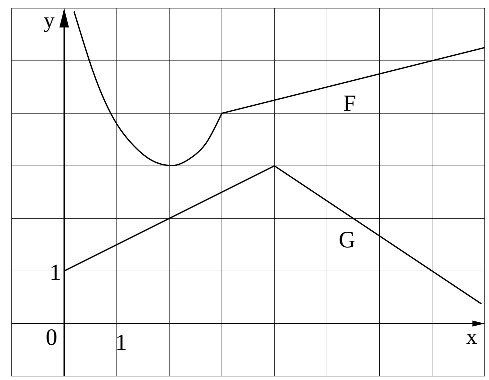
   where M is the mass of the earth, R is its radius, and G is the gravitational constant (all three are positive constants).
   (a) Find $\lim_{r\to R} F(r)$. Show you work clearly.
   (b) Determine where F is continuous.

(c) Sketch a graph of $F(r)$ for $r > 0$.
(d) Find a formula for the derivative $F'(r)$ and sketch a graph of F'.

11. §2.5: # 76 (harmonic motion)

12. (a) Find $p^{(5)}(x)$ if $p(x) = x^3 - 3x^2 + 2$
(b) Find $f^{(27)}(x)$ if $f(x) = \sin(2x)$.
(c) Find $s^{(10)}(t)$ if $s(t) = \frac{1}{t}$.
(d) How many nonzero derivatives can a polynomial of degree n have, at most?

13. Let $P(x) = F(x)G(x)$, $Q(x) = F(x)/G(x)$, and $R(x) = F(G(x))$, where F and G are the functions whose graphs are shown. Find $P'(2)$, $Q'(7)$ and $R'(2)$.



5. Applications

14. Find equations of the lines tangent and normal to the curve $y = \sqrt{x} + x$ at the point (1,2).

15. Find the points on the curve $y = \dfrac{\cos x}{2 + \sin x}$ in $[0, 2\pi]$ at which the tangent is horizontal.