TOPICS COVERED and Sample Problems
§§2.6-2.9,3.1-3.5,3.7

1. Rates of change

- Interpreting derivative as instantaneous rates of change, and find its units

1.1 The total cost (in $) of repaying a student loan at an interest rate of $r\%$ per year is $C = f(r)$.
   - (a) What is the meaning of the derivative $f'(r)$? What are its units?
   - (b) What does the statement $f'(10) = 1200$ mean?
   - (c) Is $f'(r)$ always positive or does it change sign?

1.2 A rod of length L is shown in the figure below. Suppose the mass of the rod between its left endpoint and a point at distance x from it is $m(x), 0 \leq x \leq L$, where x is measured in cm and m is measured in kilograms. In one sentence, state the meaning of the derivative $m'(x)$. What are its units?

This part of the rod has mass m(x)
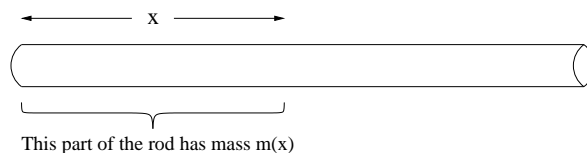
1.3 A particle moves on a straight line according to the law of motion $s = t^3 - 9t^2 + 15t + 10, t \geq 0$, where t is measured in seconds and s measures the distance in feet of the particle from a fixed point O.
   - (a) Find the velocity at time t.
   - (b) When is the particle at rest?
   - (c) When is the particle moving in the negative direction?
   - (d) When is the particle moving in the positive direction?
   - (e) Find the total distance traveled during the first 8 s.

   It may help to draw a diagram that illustrates the motion of the particle as in Figure 2, page 174.

1.4 What is acceleration when velocity is constant?

1.5 §2.7: 30 (violin string)

1.6 The cost, in dollars, of producing x yards of a certain fabric is $C(x) = 1200 + 12x - 0.1x^2 + 0.0005x^3$.
   - (a) What is the meaning of the derivative $C'(x)$?
   - (b) Find $C'(200)$. What does it predict?
   - (c) Compare $C'(200)$ with the cost of manufacturing the 201st yard of fabric, that is, $(C(201) - C(200))/(201 - 200)$

2. Related rates of change

- Given relation between dependent variables, find a relation between their derivatives
- Given a rate of change of one variable, find rate of change of another.
Need to find relation between variables first.

2.1 The pressure P, temperature T and volume V of a gas are related by the rule $T = cPV$ where c is a constant. At one instant the pressure of a gas is 30 KPa, the temperature is 18^oC, and the volume is 152 mL. If at that instant the temperature is increasing at 1^oC/hour and the volume is decreasing by 2mL/hour, at what rate is the pressure changing at that instant?

2.2 A rectangle is such that its length increases at the rate of 5 in/min and its width decreases at the rate of 7 in/min. At the moment when the length is 6 in and the width is 8 in, is the area increasing of decreasing?

2.3 §2.8: 29 (gravel pile)

2.4 §2.8: 39 (resistors)

3. Linearization

- *Approximating functions by linearization*
- *Approximating changes of functions by change in linearization*

3. §2.9: 5,7,31,35,38,40a,41

4. Finding Absolute Max/Min

- *Find abs max/min of continuous functions on closed intervals: find critical numbers*
    and compare function values at critical numbers and at endpoints
- *Find abs max/min in general: find critical numbers and get enough*
    information on increasing/decreasing or concavity to obtain a
    rough sketch of a graph that clearly illustrates absolute max/min.

4.1 Chapter 3 Review: Exercises 2, 4

4.2 Find the absolute maximum value and absolute minimum value of $f(x) = \cos^2 x - 2\sin x$ for $x \in [0, 2\pi]$.

4.3 Chapter 3 Review: Exercise 45 (length of wave with smallest velocity)

4.4 §3.1: 71 (Find max of $v(r) = k(r_0 - r)r^2$)

5. Graphing

- *Use intervals of increase/decrease, concave up/down, limiting behaviour near vertical asymptotes,*
    horizontal asymptotes, intercepts, domain, symmetry to sketch graph of function

5.1 Find domain, intervals where function is increasing/decreasing, intervals where function is concave up/down, and use behaviour at infinity if applicable, to obtain a good graph of the following functions. Clearly indicate the coordinates of all extrema and inflection points.

(a) $f(x) = x^4 + 2x^3 + 2$ (b) $f(x) = x - x^3$ (c) $f(x) = x\sqrt{6 - x}$

(d) $f(x) = x + \dfrac{1}{x^2}$ (e) $f(x) = x^2 + \dfrac{1}{x}$ (f) $f(x) = x^{1/3}(x + 4)$

5.2 §3.5: 47 (Hint: simplify the expression for the function by replacing $W/(24EI)$ by another letter, for example A.)

6. Optimization

These are word problem in which you need to identify the function that is to be maximized or mini-mized and its domain, and then use calculus to find the absolute max or min of that function on the particular domain. Please follow the outline:

> *Step 1: Draw picture. Label all variables, and all constants.*
> *Step 2: What is given? What do you want to maximize or minimize?*
> *Step 3: Find a formula for the function you want to max/min. If more than one variable*
>   *is involved, use what is known to reduce the formula to a function of one variable.*
> *Step 4: Use any method you wish to find the absolute max or min. If the function is*
>   *continuous on a closed interval, only need values at critical points and endpoints.*
>   *Otherwise, get enough information using first and/or second derivatives to obtain*
>   *a rough graph of the function that clearly illustrates its absolute max/min.*
> *Step 5: Answer the question with a sentence (in correct English).*

6.1 §3.7: 20

6.2 §3.7: 51

6.3 §3.7: 9 (maximizing yield)

6.4 §3.7: 45 (pizza)

6.5 Assume that if the price of a certain book is p dollars, then it will sell x copies where $x = 7000(1 - p/35)$. Suppose the dollar cost of producing those x copies is $15000 + 2.5x$. Finally, assume that the company will not sell this book for more than \$35. Determine the price for the book that will maximize profit.