MATH 1512 — R E V I E W E X A M 3 (Lectures 26-32)

Below is a list of the types of problems, with some examples.
Go through the homework sets 10-12.
No calculators.

DEFINITE INTEGRALS

- Approximate integrals by Riemann sums, using either the right endpoint, left endpoint, or midpoint rule. Evaluate the sum BY HAND, without using the calculator. *Example:* approximate $\int_0^\pi \sin\theta\, d\theta$ using four intervals.

- Rewrite integrals, using their definition, as limits of sums. *Example:* Write $\int_0^1 x\, dx$ as the limit of a sum (this is the definition of the integral). Answer: $\lim_{n\to\infty} \sum_{j=1}^n x_j \Delta x$ where $\{x_j\}, \Delta x$ is a uniform partition of $[0, \pi]$.

- Recognize when the limit of a sum is an integral and rewrite it as such. Then evaluate using the fundamental theorem. *Examples:* Find $\lim_{n\to\infty} \sum_{j=1}^n \sin(x_j)\Delta x$ where $\{x_j\}, \Delta x$ is a uniform partition of $[0, \pi]$. Write $\lim_{n\to\infty} \sum_{j=1}^n \sqrt{1 + f'(x_j)}\Delta x$, where $\{x_j\}$ is a uniform partition of $[a, b]$, as an integral.

- Evaluate integrals by interpreting them as areas. *Examples:* Find $\int_0^2 2 - 3\sqrt{4 - x^2}\, dx$. Find $\int_1^4 |x - 2|\, dx$. Find $\int_0^2 H(x - 1)\, dx$ where H is the Heaviside function. Homework.

- State the fundamental theorem, both parts 1 and 2.

- Use the fundamental theorem (Part 1) to differentiate functions defined as integrals. *Examples:* §4.3: 9,18,60. Chapter 4 Review: 8, 58ab, 59, 60. Homework.

- Use the fundamental theorem (Part 2) to evaluate integrals, using substitution if necessary. *Examples:* Chapter 4 Review: odd 11-31.

- If a quantity is given as an integral of a physical variable that has units, state the units of the integral. If a quantity is given as the derivative of a physical variable that has units, state the units of the derivative.

- Understand the function $g(x) = \int_a^x f(t)\, dt$. *Example:* Given graph of f, can you sketch graph of g? What is g'? Where is g increasing? decreasing? Differentiate $g(x) = \int_a^u f(t)\, dt$, where u is some function of x, $u = u(x)$, using the chain rule.

- **You do not need to:** compute integrals using the definition

APPLICATIONS

- Given a rate of change of a quantity (such as velocity, flow rate, growth rate) find changes in the quantity. *Examples:* many examples in HW 12. *Examples:* Chapter 4, Review: Concept Check 5,6. Exercise 53-55,

- Given a rate of change of a quantity and an initial value, find the quantity by solving the initial value problem.

OTHERS

- Indefinite integrals
- Concept questions:

  What is an initial value problem?

  How can you interpret the definite integral $\int_a^b Q'(x)\,dx$?

  Is $\int_a^b Q(x)\,dx = \int_a^b Q(u)\,du$?

  How do you integrate functions defined piecewise? Example: §3.9: 55

ANTIDERIVATIVES/ DIFFERENTIAL EQUATIONS

- Find antiderivatives of $f(\xi)$, also written as $\int f(\xi)\,d\xi$. *Examples* : §3.9: 5, 13.
- Solve differential equations of form $y' = f(x)$, or $y'' = f(x)$, w/ or w/o initial conditions. *Examples* : §3.9: 33, 40, 43.
- Applications: Given acceleration, find velocity, distance travelled. Given linear density, find mass. Given a rate of change of a quantity, find information about the quantity.