NAME: MATH 1512 – HW 2 Due: Sunday Jan 29, 2023

HOMEWORK DAY 2 – *The limit of a function. Infinite limits. §1.5*

1. §1.5: 1

2. §1.5: 4. (a) (b) (c)

   (d) (e) (f)

3. §1.5: 9. Add: (g) $\lim_{x\to 6} f(x)$.

   (a) (b) (c) (d) (e)

   (f) (g)

4. Find the following limits. Follow the notation in the worked out example for all infinite limits. When a limit does not exist, briefly explain why.

   (a) §1.5: 27

   (b) §1.5: 28

$$\lim_{x\to 5^-} \frac{x+1}{x-5} \left(\frac{6}{0^-}\right) = -\infty$$

(c) §1.5: 29

(d) §1.5: 30

(e) §1.5: 31

(f) §1.5: 32

(g) $\lim_{x \to 1^-} \frac{1}{x^3 - 1}$

(h) $\lim_{x \to 1^+} \frac{1}{x^3 - 1}$

(i) $\lim_{x \to 1} \frac{1}{x^3 - 1}$

5. §1.5: 11

6. §1.5: 16

7. Let $f(x) = \begin{cases} 1+x & \text{if} \quad x < -1 \\ x^2 & \text{if} \quad -1 \le x < 1 \\ 2-x & \text{if} \quad 1 < x \le 3 \\ -1 & \text{if} \quad x > 3 \end{cases}$

(a) Sketch a graph of the function.

(b) Find the following limits or determine they do not exist (if so, explain why not).

$\lim_{x \to -2} f(x) =$

$\lim_{x \to -1} f(x) =$

$\lim_{x \to 1} f(x) =$

$\lim_{x \to 3} f(x) =$

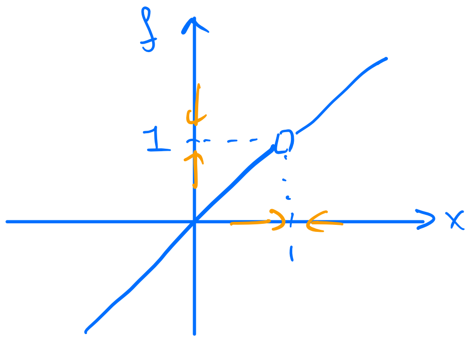
(c) State the values of all a for which $\lim_{x \to a} f(x)$ exists.

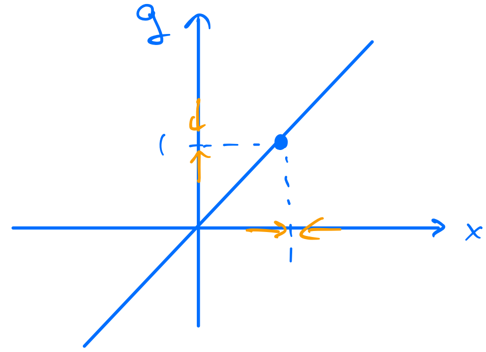
8. Consider the two functions $f(x) = \frac{x^2-x}{x-1}$ and $g(x) = x$.

(a) Are the two functions equal? Explain.

No. The domain differ. Domain of f: $x \neq 1$
Domain of g: all $x \in \mathbb{R}$

(b) Sketch a graph of both functions.





(c) Use your sketch to find $\lim_{x \to 1} f(x)$ and $\lim_{x \to 1} g(x)$. Illustrate the limit in your sketch.

$\lim_{x \to 1} f = \lim_{x \to 1} g = 1$

9. Consider the function $g_1(x) = \dfrac{x^2 - 4}{x - 2}$.

(a) Sketch the graph of $g_1(x)$ and of $g_2(x) = x + 2$.

(b) Find $\lim_{x \to 2} g_1(x)$ and $\lim_{x \to 2} g_2(x)$

(c) Explain why the limit in (b) is the slope of tangent line of $f(x) = x^2$ at $x = 2$. Illustrate with a figure.

10. Let $f(x) = \sin(x)$.

(a) Sketch a clearly labeled graph of $f(x)$, using a 1-1 scale.

(b) Explain why the slope of the tangent line to the graph of f at the origin ($x = 0$) is given by the limit

$$\lim_{x \to 0} \frac{\sin x}{x} \ .$$

(c) Approximate the limit using a table of values.

(d) Use your result in (c) to find an equation for the tangent line to f at the origin, and add a graph of it to your sketch in (a).

11. Find the following finite or infinite limits. If the limit does not exist, explain why not. Follow the worked out example.

(a) $\lim_{x\to -2}(2x + x^2)$ $= -4 + 4 = 0$

(b) $\lim_{h\to 1}(2 - h/2)$

(c) $\lim_{x\to 1^+} \frac{1}{x-1}$

(d) $\lim_{x\to -3^-} \frac{x+2}{x+3}$

(e) $\lim_{x\to 1} \frac{1}{x-1}$

(f) $\lim_{x\to 1} \frac{1-2x}{(x-1)^2}$

(g) $\lim_{x \to \pi} \frac{1 + \cos x}{1 - x}$

(h) $\lim_{x \to \pi} \frac{1 - x}{1 + \cos x}$

(i) $\lim_{t \to 1} \frac{1 - t^2}{1 - t}$ $= \lim_{t \to 1} \frac{(1-t)(1+t)}{1-t} = \lim_{t \to 1} (1+t) = 2$

since $t \neq 1$
can divide by $1-t$

(j) $\lim_{x \to 0} \frac{\sqrt{x+4} - 2}{x}$

(k) $\lim_{x \to 1} \frac{(1 + x)^3 - 1}{x}$

(l) $\lim_{x\to 0} \frac{(1+x)^3 - 1}{x}$

(m) §1.6: 13

(n) §1.6: 17

(o) §1.6: 19

(p) §1.6: 21

(q) §1.6: 23

(r) §1.6: 25

(s) §1.6: 27

12. §1.6: 38 (Squeeze Theorem)

13. §1.6: 41 (Squeeze Theorem)

HOMEWORK DAY 4 – *Continuity §1.8*

§1.8: 1.

§1.8: 3.

§1.8: 5.

§1.8: 6.

14. §1.8: 17

15. §1.8: 20

16. §1.8: 22

17. §1.8: 23

18. §1.8: 42. Also sketch a graph of f.

19. §1.8: 44

20. §1.8: 46 (gravitational force) Also sketch a graph of $F(r)$.

21. §1.8: 47 (for what c is f continuous everywhere?) Also sketch a graph of $f(x)$ for the value of c that you found.