HOMEWORK DAY 5 – *Definition of the derivative §2.1*

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1. A particle moving along a horizontal line has position $s(t)$, where s is measured in meters, t in seconds.

   (a) What is the meaning of the derivative $s'(t)$? What are its units?

   (b) What is the meaning of the derivative $s''(t) = (s')'(t)$? What are its units?

2. §2.1: 3 (quadratic, find slope in two ways)

3. §2.1: 10 ($f = 2\sqrt{x}$)

4. §2.1: 15 (given graphs of f, plot f')

5. §2.1: 16 (two runners)

6. §2.1: 17 (arrange numbers)

7. A cylindrical tank initially holds 100,000 gallons and is being drained from the bottom over the interval of 1 hour. The amount of gallons of water V remaining in the tank after t minutes is given by

$$V(t) = 100,000(1 - \frac{t}{60})^2 , \quad t \in [0, 60] .$$

(a) What is the meaning of the derivative $V'(t)$? What are its units?

(b) In practical terms, what does it mean to say that $V'(10) \approx -2777.8$?

(c) Sketch a graph of $V(t)$. What is a geometrical meaning of the derivative $V'(t)$?

(d) For $t > 0$, is $V'(t) \geq 0$ or ≤ 0? Use only your graph above

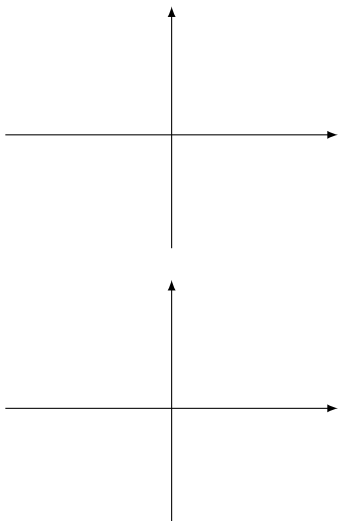(e) At what time is $|V'(t)|$ largest? Explain physically why that makes sense.

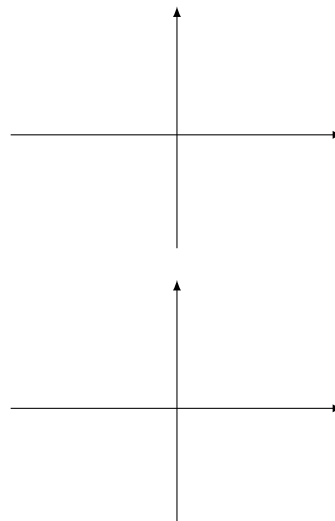8. Let $f(x) = x^3$. Find $f'(a)$

9. §2.1: 25 $(f = 1/(t^2 + 1))$

10. §2.2: 3

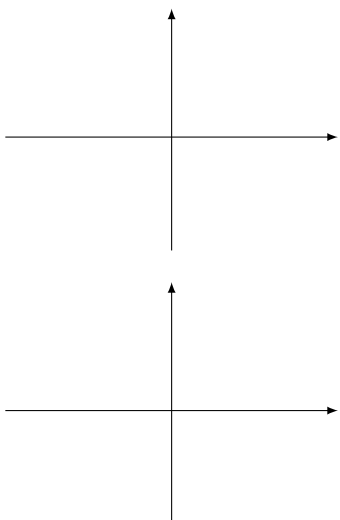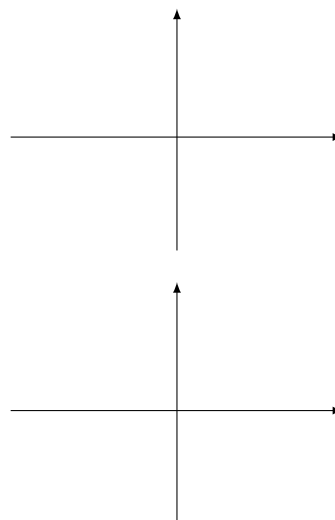11. §2.2: 4

§2.2: 5

12. §2.2: 8

§2.2: 9

§2.2: 46. First answer: which is f, which is f'? Then answer question.

13. §2.2: 14

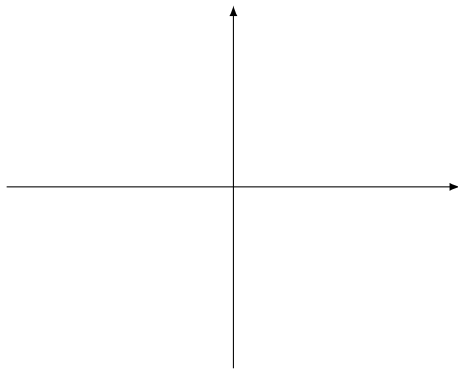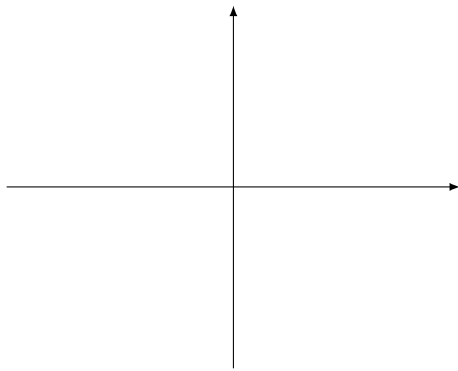14. §2.2: 35 (trees) What are the units for H'(x)? When do the trees grow the fastest?

15. §2.2: 40 (differentiability)

For the problems below, use the definition $f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$.

16. §2.2: 28 (Hint: Ask your instructor or TA to show you Pascal's triangle)

17. Let $f(x) = x + 1/x$.

    (a) Use superposition of the graphs of x and $1/x$ to sketch the graph of f.

    (b) Below your graph in (a), in the space given above, sketch the graph of f'.

(c) Use the definition to find the formula for f'

(d) Use superposition of graphs to sketch the graph of f' and check that it agrees with your graph in b.

18. Let $f(x) = \sqrt{x}$. Use the definition to find the derivative $f'(x)$ for $x > 0$. (Hint: multiply top and bottom of the quotient by $\sqrt{x+h} + \sqrt{x}$). Sketch the graph of f and f'.

Give two reasons why the function is not differentiable at $x = 0$.

19. Let $f(x) = x^{1/3}$. Use the definition to find the derivative $f'(x)$ for $x \neq 0$. (Hint: multiply top and bottom of the quotient by $(x+h)^{2/3} + (x+h)^{1/3}x^{1/3} + x^{2/3}$). Sketch the graph of f and f'.

Give one reason why the function is not differentiable at $x = 0$.

Differentiate the following functions. Simplify your answer.

20. §2.3: 1

21. §2.3: 3

22. §2.3: 4

23. §2.3: 6

24. §2.3: 7

25. §2.3: 9   *Answer* : $s(t) = t^{-1} + t^{-2}$, $s'(t) = -t^{-2} - 2t^{-3} = -\frac{1}{t^2} - \frac{2}{t^3} = -\frac{t+2}{t^3}$ (last step is optional)

26. §2.3: 10

27. §2.3: 11

28. §2.3: 12

29. §2.3: 13

30. §2.3: 34

31. §2.3: 35

32. §2.3: 39

33. §2.3: 43

34. §2.3: 46

35. §2.3: 27

36. §2.3: 28

37. §2.3: 79(a,b)

38. §2.3: 84

39. §2.3: 86(a,b)