NAME: MATH 1512 – HW 6 Due: Sunday Feb 26, 2023

HOMEWORK DAY 14 – *Related Rates §2.8*

1. §2.8: 2 (circular oil spill)

2. §2.8: 13 (plane)

3. §2.8: 14 (snowball)

4. §2.8: 16 (two ships) (Answer: ...)

5. §2.8: 29 (growing pile of gravel)

6. §2.8: 39 (two resistors connected in parallel) (Answer: ...)

HOMEWORK DAY 15 – *Linear Approximations §2.9*

Please avoid using differentials here. All problems can be done using either

$$f(x) \approx L(x) = f(a) + f'(a)(x - a)$$

or

$$\Delta f = f(a + h) - f(a) \approx \Delta L = f'(a)h$$

7. §2.9: 1

8. §2.9: 5 (approximate $\sqrt{1-x}$ for small x)

9. **Linear approximations of functions.** Show that for any real number k, $(1 + x)^k \approx 1 + kx$ for small x. Estimate $(1.02)^{0.7}$ and $(0.9)^{-0.3}$.

10. **Linear Extrapolation.** The Table in §2.7: # 27, shows the world population every 10 years since 1900.

    (a) Use the last two entries to estimate the slope of the function $P(t)$ at $t = 2010$ (ie, the rate of change or the population with respect to time).

    (b) Use your result in (a) to find the linear approximation of $P(t)$ at $t = 2010$.

    (c) Use the linear approximation to estimate the population at $t = 2020$.

    (d) Go online and find the actual population in 2020 and compare with your estimate in (c). Is your estimate an under- or over-estimate?

11. **Approximating measurement errors.** The circumference c of a sphere was measured to be 84 cm with a possible error of at most 0.5 cm (in absolute value). Approximate the maximum error using this measurement of c to compute

(a) the surface area of the sphere

(b) the volume of the sphere
(Hint: in (a) the first step is to write surface area in terms of c. Then write an approximation of ΔS in terms of Δc. Similarly for (b).)
Also compute the exact maximum error and compare to your approximation.
Note: In practice, the approximation is much easier to obtain and is sufficiently good, since measurement errors are not precise in the first place.

12. **Approximating changes.** Approximate the amount of paint needed to apply a coat of paint 0.05cm thick to a hemispherical dome with diameter 50 m. Use linear approximations. Also compare the exact amount of paint needed and compare to the linear approximation. In practice, the coat of paint will not be precisely 0.05cm thick so the approximation gives a good estimate.

13. **Approximating changes.** Approximate the volume of a thin cylindrical shell with height h, inner radius r, and thickness Δr. Use linear approximations. Also find the exact volume and compare.

14. (a) Give an example of a function that is bounded below on its domain but has no absolute minimum. Explain.

    (b) Give an example of a function that is bounded above on its domain but has no absolute maximum. Explain.

15. Sketch the graph of the following functions and use your sketch to determine the absolute maximum and minimum values of f, if they exist.

    (a) §3.1: 15 (line)

(b) §3.1: 17 (1/x)

(c) $f(t) = \cos(3\pi t)/2 + 1$

(d) §3.1: 27 (piecewise)

(e) $f(x) = \sqrt{r^2 - x^2}$. Hint: to graph it, first find the domain of f, then note that $y = \sqrt{r^2 - x^2} \leftrightarrow y^2 = r^2 - x^2, y \geq 0$ so $x^2 + y^2 = r^2$, $y \geq 0$.

16. Find the absolute maxima and minima of the following functions, clearly explaining your reasoning. (Use the Closed Interval Method)

(a) $f(x) = 2x^3 - 3x^2 - 12x + 1 \ , \quad -2 \leq x \leq 3$

(b) $f(x) = x - 2\cos x\ , \quad 0 \leq x \leq 2$

(c) $f(x) = x^2(x^2 - 1)\ , \quad x \in [-1, 2]$

(d) $f(t) = \sqrt[3]{t}(8-t)$, $\quad t \in [0, 8]$.

(e) $f(t) = 2\cos t + \sin(2t)$, $\quad 0 \leq t \leq \pi/2$