NAME: MATH 1512 – HW 7 Due: Sunday Mar 5, 2023

HOMEWORK DAY 17 – *Derivatives and the shape of the graph §3.3*

1. §3.3: 1

2. §3.3: 8

3. §3.3: 10

4. For the function $f(x) = 3x^4 - 4x^3 + 2$

   (a) Find the intervals where the function is increasing/decreasing and the coordinates of all local max/min.

   (b) Find the intervals where the function is concave up/down and the coordinates at all inflection points.

(c) Use the above information to sketch a rough graph of the function.

5. For the function $f(x) = \cos^2 x - 2\sin x,\ x \in [0, 2\pi]$

   (a) Find the intervals where the function is increasing/decreasing and the coordinates of all local max/min.

(b) Find the intervals where the function is concave up/down and the coordinates at all inflection points.

(c) Use the above information to sketch a rough graph of the function.

6. §3.4: 4

7. Find the following limits. Carefully show your work.

   (a) §3.4: 9

   (b) §3.4: 11

   (c) §3.4: 12

8. For the function $f(x) = \dfrac{2}{x^2 + 4}$

(a) Find the intervals where the function is increasing/decreasing and the coordinates of all local max/min.

(b) Find the intervals where the function is concave up/down and the coordinates at all inflection points.

(c) Find the limiting behaviour as x approaches $\pm\infty$.

(d) Use the above information to sketch a rough graph of the function. Include all intercepts on graph.

9. Let $f(x) = \dfrac{4 - x}{3 + x}$.

(a) Find the intercepts of f and all its asymptotes.
Find the limiting behaviour near each vertical asymptotes ($\lim_{x \to a^{\pm}} f(x)$).
Use this information to sketch a guess for the graph of f.

(b) Confirm details of your guess by finding the intervals where the function is increasing/decreasing and concave up/down. Clearly mark any local extrema and inflection points in your plot.

(c) Use algebra (long division in this case) to rewrite the function as a polynomial plus a proper quotient. Could you have obtained the graph of f from this result?

10. Consider the function

$$f(x) = \frac{\sin x}{x}$$

(a) State the domain and all intercepts of the function.

(b) Show that f is even.

(c) Find all asymptotes of the function.

(d) Explain why the following limit equals the derivative of $g(x) = \sin x$ at $x = 0$ and use that to deduce that

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

(e) Sketch a graph of the function that reflects all the information you found above.

11. The reaction rate V of a common enzyme reaction is given in terms of substrate level S by

$$V = \frac{V_o S}{K + S}, \quad S \geq 0$$

where V_o and K are positive constants.

(a) Show that V is an increasing function of S. Explain why it follows that V has no absolute maximum value.

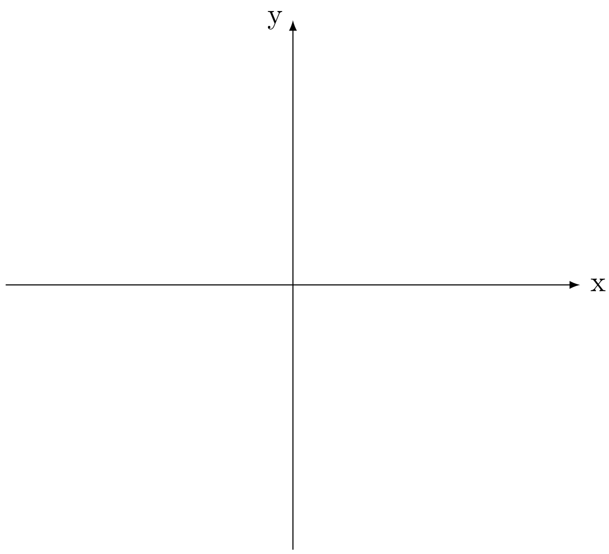
(b) What is $\lim_{S \to \infty} V$?

(c) Determine the concavity of the graph of V (where is it concave up? where concave down?).

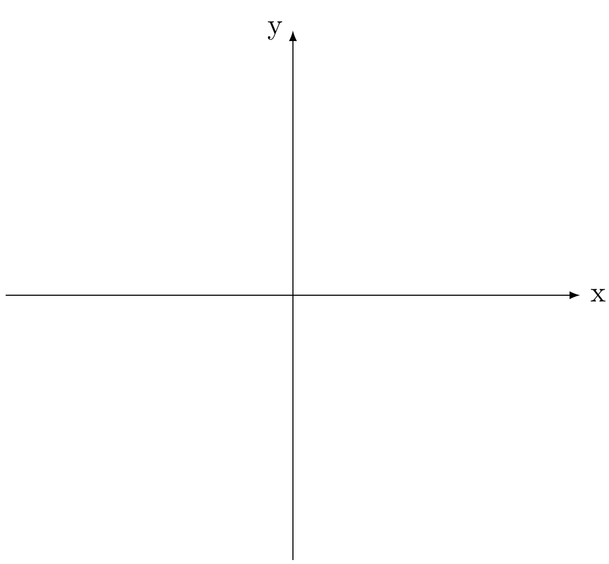
(d) Sketch a graph of V as a function of S that reflects all the information you found above.

12. For each of the following polynomial functions:
    (1) Graph the function using only the roots, multiplicity of the roots, and the behaviour as $x \to \pm\infty$.
    (2) Find the coordinates of the local max/min and add them to the plot.
    (3) Find the coordinates of the inflection points and add them to the plot.
    (4) If the function is odd or even, make a note of it.

    (a) $f(x) = x^4 - 8x^2$

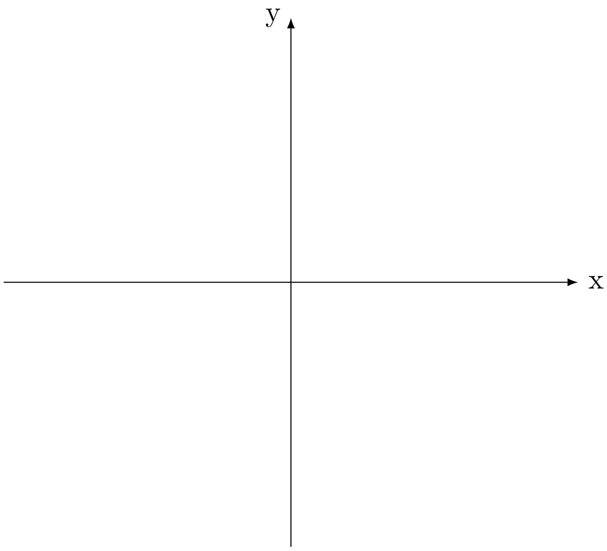


    (b) $f(x) = 3x^2 - x^3$

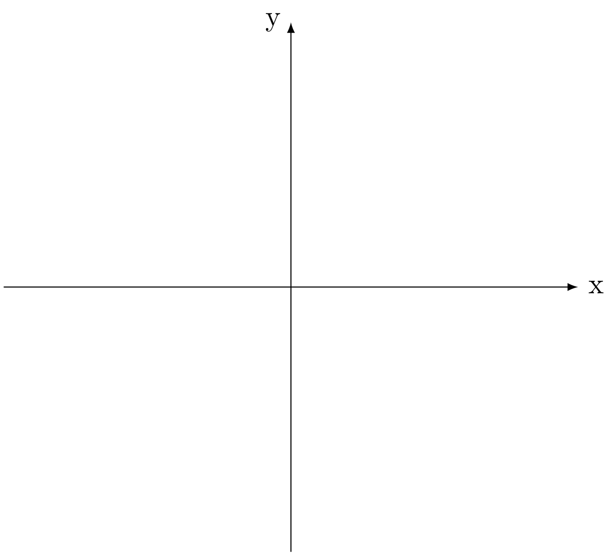


13. For the following rational functions:
(1) First sketch a rough graph of the function using superposition of simple functions.
(2) Find the coordinates of the local max/min using f' and add them to the plot.
(3) Find the coordinates of all inflection points using f'' and add them to the plot.
In addition, if the function is odd or even, make a note and ensure your graph reflects it.

(a) $f(x) = x + \dfrac{1}{x}\ , \quad x > 0$



(b) $f(x) = x + \dfrac{1}{x^2}\ , \quad x > 0$



14. According to a mathematical model, the velocity v of the airstream in a small circular pipe (here, the trachea) is related to the radius r_0 of the pipe by

$$v(r) = k(r_0 - r)r^2 \ , \quad \frac{r_0}{2} \leq r \leq r_0$$

where $k > 0$ and r is the radial distance from the center of the pipe. (The model is not good for $r < r_0/2$.)

(a) Sketch a graph of this cubic polynomial using solely the multiplicity of each root and the limiting behaviour as $r \to \pm\infty$. Sketch the polynomial for all r as a dashed line, then highlight the restricted domain by a solid line.

(b) Find the absolute maximum value of v on the interval $[\frac{r_0}{2}, r_0]$. Add the corresponding point(s) into your graph in (a).

15. Solve the following inequalities $f(x) > a$ or $< a$ by (1) first sketching a graph of the function (using roots and multiplicity in case of polynomials) and the line $y = a$ (including intercepts), then (2) finding the interval in question.

    (a) $2 - x^2 > 0$

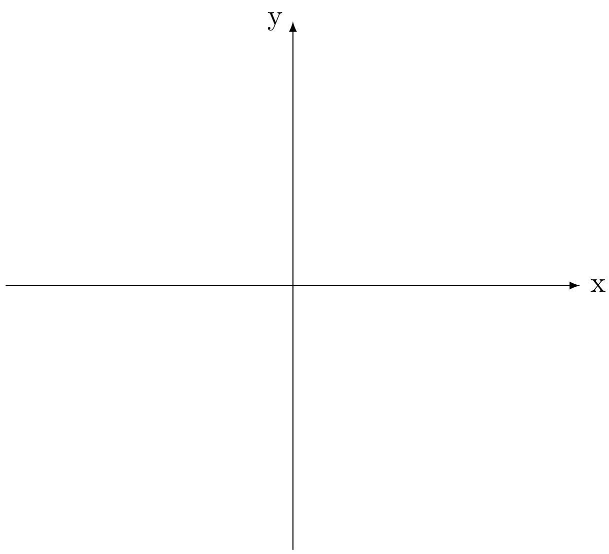
    (b) $x^3 - x^2 < 0$

    (c) $\sin x > 1/2, \quad x \in [0, 2\pi]$

    (d) $\tan x > 1, \quad x \in (-\pi/2, \pi/2)$

16. For each of these problems, sketch a graph clearly showing domain, asymptotes, intercepts, local maxima/minima and inflection points.

(a) $f(x) = 2x^3 - 3x^2 - 12x + 2$



(b) $f(x) = \dfrac{x}{x^2 + 9}$