NAME: MATH 1512 – HW 9 Due: Sunday Mar 26, 2023

HOMEWORK DAY 24 – *Antiderivatives (§3.9)*

1. Find the most general antiderivative for the following functions

   (a) $f(x) = 6x^5 - 8x^4 - 9x^2 - 3$

   (b) $f(x) = 3x^4 - \frac{2}{3}x^3 + \pi x + \sqrt{2}$

   (c) $f(x) = 4x^{-2/3} - 2x^{5/3}, \quad x > 0$

   (d) $f(x) = (x-5)^2$

   (e) $f(x) = \sin(4x)$

(f) $f(t) = \sqrt[4]{t} + \sqrt[4]{x}, \quad t, x > 0$

(g) $f(s) = \dfrac{4 - 2s + \sqrt{s}}{s^{1/2}}, \quad s > 0$

2. Find f.

(a) $f'(x) = \sqrt{x} - 2\ , \quad f(9) = 4$

(b) $f'(x) = 5x^{2/3}\ , \quad f(8) = 21$

(c) $f'(t) = t + \dfrac{1}{t^3}\ , \quad t > 0\ , \quad f(1) = 6$

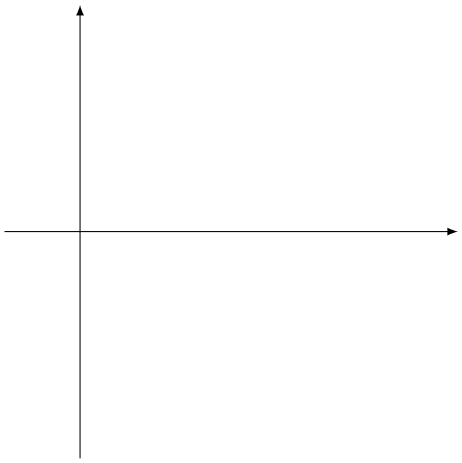
(d) $f'(\theta) = \sec\theta(\sec\theta + \tan\theta)\ , \quad f(\pi/4) = 1, \quad \theta \in (-\pi/2, \pi/2)$

(e) $f''(x) = 8x^3 + 5\ , \quad f(1) = 0\ , \quad f'(1) = 8$

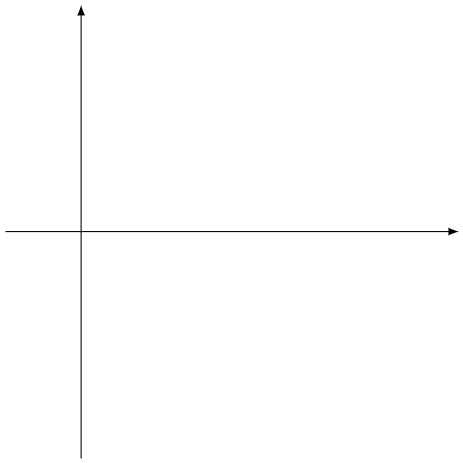
(f) $f''(t) = \cos t + \sin t\ , \quad f(0) = 3\ , \quad f'(0) = 4$

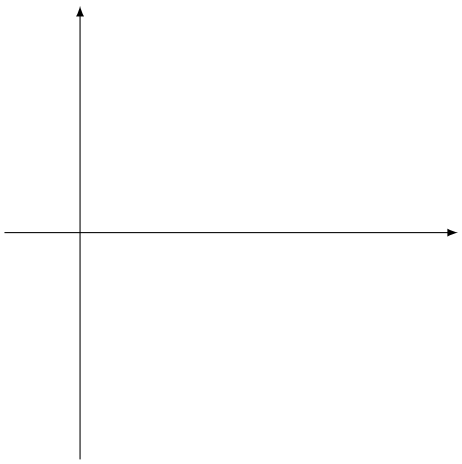
3. Draw the plots of the two functions one below the other. (Label all axes.)

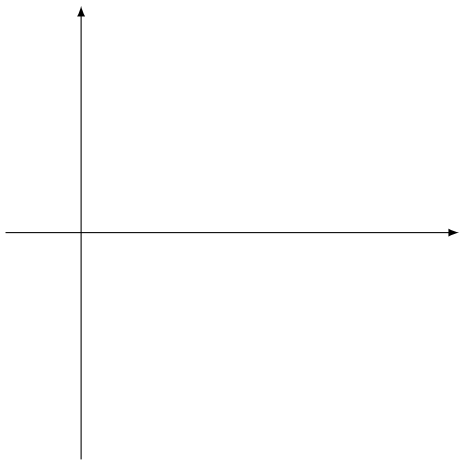
(a) §3.9: 53



(b) 3.9: 55







4. §4.1: 2

5. Use the left-endpoint rule with $n = 6$ to approximate the area under $y = \sin x$ between $x = 0$ and $x = \pi$. Simplify your answer as much as possible without using a calculator. Draw a diagram that illustrates what the sum represents.

6. §4.1: 13

7. (a) Sketch a graph of $f(x) = \dfrac{1}{1+x^2}$

(b) Use your graph to show that the area A under the graph, over the interval $x \in [-2, 2]$, is bounded above and below by

$$\frac{4}{5} < A < 4$$

(c) Find better upper and lower bounds for A.

8. Evaluate the following sums.

(a) $\displaystyle\sum_{k=2}^{5} \frac{2k}{k-1}$

(b) $\displaystyle\sum_{j=0}^{5} j^2 \sin(j\pi/6)$

(c) $\displaystyle\sum_{k=3}^{100} 2$

(d) $\displaystyle\sum_{j=1}^{1000} j$

9. Use summation notation to express the sums

(a) $\frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}$

(b) $3 + 6 + 9 + 12 + 15$

(c) $2/5 + 3/7 + 4/9 + 5/11$

10. Which is larger, $\sum_{j=1}^{N} j^2$ or $\sum_{j=1}^{N^2} j$? Explain why.