NAME: MATH 1512 – HW 10 Due: Sunday Apr 2, 2023

HOMEWORK DAY 26 – – *Definition of definite integral. Properties. (4.2)*

1. *The definite integral is the limit of a Riemann sum.*

   (a) §4.2: 20 (express limit as a definite integral)

   (b) §4.2: 26 (express definite integral as a limit of a Riemann sum)

   (c) §4.2: 84 (express limit as a definite integral)

2. *Approximating and evaluating the limit.* Consider the integral $\int_0^3 x^2\,dx$.

   (a) Use the right endpoint rule with $n = 6$ to approximate the integral. Round the answer to four decimal places. (Include a graph)

(b) Express the integral as the limit of a Riemann sum.

(c) Evaluate the limit in (b).

3. *Interpreting the definite integral in terms of areas.*

(a) Consider the function $g(x)$ given in §4.2: 36. Evaluate

i. $\int_0^2 g(x)\,dx =$

ii. $\int_2^6 g(x)\,dx =$

iii. $\int_0^7 g(x)\,dx =$

iv. $\int_0^7 |g(x)|\,dx =$

(b) §4.2: 42

(c) §4.2: 44

(d) §4.2: 46

(e) §4.2: 63

4. Using the properties of the definite integral.

   (a) §4.2: 51

   (b) §4.2: 52

   (c) §4.2: 57

   (d) §4.2: 58

5. The **Heaviside function** H is defined by $H(t) = \begin{cases} 0\ , & t < 0 \\ 1\ , & t \geq 0 \end{cases}$

(a) Sketch $H(t)$. Find $\displaystyle\int_{-2}^{4} H(t)\, dt$.

(b) Sketch $H(t-2)$. Find $\displaystyle\int_{-2}^{4} H(t-2)\, dt$.

(c) Sketch $H(t-2) + H(t)$ (use superposition of graphs).

Use the above to find $\displaystyle\int_{-2}^{4} [H(t-2) + H(t)]\, dt =$

6. §4.3: 4 (verify Theorem for simple example)

7. Find $g'(x)$ where

   (a) $g(x) = \int_1^x \cos(t^2)\,dt$

   (b) $g(x) = \int_x^3 t^3 \sin t\,dt$

(c) $g(x) = \int_{-1}^{x^2} e^{t^2}\, dt$

(d) $g(x) = \int_{-1/x}^{x} \sqrt{1+t}\, dt$

(e) $g(x) = \sin(-x^2) \int_{0}^{x^2} e^{t^2}\, dt.$

8. Evaluate the following definite integrals. All of these can be done by inspection. For example, to find $\int \sec^2(\pi x)\,dx$ you may guess the antiderivative to be $\tan(\pi x)$, but $\frac{d}{dx}[\tan(\pi x)] = \pi\sec^2(\pi x)$. So the correct antiderivative is $\int \sec^2(\pi x)\,dx \frac{1}{\pi}\tan(\pi x) + C$. For these simple problems, always check the derivative of the antiderivative you found.

   (a) $\displaystyle\int_1^3 (9x^2 + 2x - 4)\,dx$

   (b) $\displaystyle\int_0^4 (t^2 + t^{3/2})\,dt$

   (c) $\displaystyle\int_1^4 \frac{x-1}{\sqrt{x}}\,dx$

(d) $\displaystyle\int_0^2 (y-1)(2y+1)\,dy$

(e) $\displaystyle\int_1^2 \frac{s^4+1}{s^2}\,ds$

(f) $\displaystyle\int_1^9 \sqrt{3x}\,dx$

(g) $\int_0^{\pi/2} \sin(2x)\,dx$

(h) $\int_{\pi/3}^{\pi/2} \cos(x/2)\,dx$

(i) $\int_{-2}^{2} x^2 \sin(x)\,dx$

(j) $\displaystyle\int_0^{\pi/8} \sec^2(2\theta)\,d\theta$

(k) $\displaystyle\int_{-2}^{2} f(x)\,dx$ where $f(x) = \begin{cases} 2 & \text{if } -2 \le x \le 0 \\ 4 - x^2 & \text{if } \ 0 < x \le 2 \end{cases}$

(l) $\displaystyle\int_0^{\pi} f(x)\,dx$ where $f(x) = \begin{cases} \cos x & \text{if } \ 0 \le x \le \pi/2 \\ \sin x & \text{if } \ \pi/2 < x \le \pi \end{cases}$