**BASICS**

- **Graphs.** Be able to graph basic functions, such as polynomials (eg, $f(x) = x^3 - x$, $x^2 + ax + b$, $x(x-1)^2(x+2)^3$, know about the effect of multiplicity of roots on derivatives), simple rational functions (eg, $f(x) = 1/x, 1/x^2, x + 1/x$), roots ($\sqrt{x}$, $\sqrt[3]{x}$), trig functions ($\sin x, \cos x, \tan x, \sec x$) and their translations and transformations.
- **Algebra.** Be comfortable manipulating quotients, powers, absolute values. Solve equalities and inequalities.
- **Derivatives.** Use power rule, product rule, quotient rule, chain rule, to find derivatives.
- **Integration.** Find antiderivatives (when possible) either directly or using substitution. Always check your answer!!!

**LIMITS, CONTINUITY, DIFFERENTIABILITY**

- **Computing Limits.**
  - Find limits as $x \to c$ and onesided limits as $x \to c^{\pm}$ of continuous functions, functions whose graphs have holes, functions defined piecewise.
  - Find limits as $x \to \pm\infty$ of polynomials, rational functions, other (eg $\frac{1}{\sqrt{x}}$, $\frac{x-3}{\sqrt{1+2x^2}}$), and horizontal asymptotes if applicable.
  - Find vertical asymptotes and limiting behaviour from either side.
  - Recognize when a limit is a derivative.
- **Definition of continuity.** State the definition of continuity. Determine whether a function is continuous at a point either using the rules (theorem 7 on page 102), or using the definition, specially at breakpoints of functions defined piecewise.
- **Definition of differentiability.** State the definition of differentiability. (What does it mean to say "$f(x)$ is differentiable at $x = a$"?) What is the relation between differentiability and continuity? Give examples of functions that are continuous but not differentiable at a point.
- **Definition of the derivative.** State the definition of the derivative. Be able to compute derivatives using the definition. Be able to compute derivatives using the rules, include derivatives of functions defined piecewise.

**DERIVATIVES AND THEIR APPLICATIONS**

- **Geometric interpretation of the derivative,** as the slope of tangent to a curve at a point, or its curvature. Find equations of tangent lines. Sketch graph of f' and f" given graph of f.
- **Rate-of-change interpretation of the derivative.** Interpret derivative as instantaneous rate of change of a quantity $Q$ with respect to a variable $x$. It is approximately equal to $\Delta Q/\Delta x$, for small $\Delta x$. Units of derivatives. (Some applications: position, velocity, acceleration. Population growth. Density of rod=rate of change of mass with respect to length.)
- **Rules of differentiation.** Compute derivatives using product rule, quotient rule, chain rule, implicit differentiation. Include derivatives of $x^p$, $\sin(x)$, $\cos(x)$, $\tan(x)$, $\sec(x)$
- **Related Rates.** Given a relation between quantities, find a relation between their derivatives.
- **Graphing.** Use derivatives to find intervals where the function is increasing/decreasing, intervals where the function is concave up/down, local maxima and minima, and inflection points, Combine with intercepts, limiting behaviour at endpoints to obtain graph of function, incl coordinates of local extrema and inflection points.

- **Linearization.**
  - Find the linearization $L(x)$ of a function $f(x)$ at a base point $x = a$. Use it to approximate $f(x) \approx L(x)$.
  - Approximate the change of a function $\Delta f$ near a base point $x = a$ by the change in its linearization, $\Delta L = f'(a)\Delta x$. Equivalently, $\Delta f \approx f'(a)\Delta x$.
- **Optimization.**
  - Find absolute maxima/minima for continuous functions on bounded domains.
  - Find all absolute and local maxima/minima for any given function. JUSTIFY your answer (either by graph, or using first derivative test or second derivative test) Note: the fact that the function has a local minimum at one point is not a justification for absolute minimum.
  - Solve word problems.

INDEFINITE INTEGRALS

- **Antiderivatives.** Find antiderivatives. Simplify first if appropriate.
- **Differential equations and Initial Value Problems.** Solve first and second order differential equations of the form $y' = f(x)$ or $y" = f(x)$ with or without initial conditions. Application: Given velocity or acceleration, plus initial conditions, find position of particle.

DEFINITE INTEGRALS

- **Definition of the definite integral.** State the definition as a limit of a sum. Be able to rewrite the limit of a sum as an integral, when applicable.
- **Evaluate or approximate definite integrals.**
  - Using geometry, ie, by interpreting them as areas (positive, negative, or differences of).
  - Using fundamental theorem of calculus
  - Approximate definite integrals by rectangles.
- **Understand the function $g(x) = \int_a^x f(t)\,dt$.**
  - Given graph of $f$, can you sketch graph of $g$?
  - What is $g'$?
  - Where is $g$ increasing? decreasing? concave up? down?
  - Differentiate $g(x) = \int_a^u f(t)\,dt$, where $u$ is some function of $x$, $u = u(x)$, using the chain rule.
- **Average value of a function over an integral.** Be able to compute $f_{av}$. Understand geometric interpretation of $f_{av}$.
- **Interpret the definite integral.**
  - As the limit of a sum (definition).
  - As an area (or difference of areas).
  - As an average.
  - As a total change (of the antiderivative). Application: given velocity, find displacement, and total distance travelled.
- **Applications.**
  - Express areas between curves as an integral.
  - Express volumes of solids of revolution as an integral.
  - Express lengths of curves as an integral (understand the derivation!).
  - Given a force, express work as an integral.

## REVIEW PROBLEMS

To pass this class, you absolutely must be able to differentiate correctly, including implicit differentiation, and compute definite and indefinite integrals, including substitution. Good review problems for these are those in Chapter 2 Review (13-44) and Chapter 4 Review (9-28), §4.5: odd 5-29. All odd ones have solutions in the back of the book. Do as many as you need to be solid in these skills.

Below is a sample set of additional review problems. Many of them are previous HW or Mixed Review problems. All HW and mixed review are good exam problems.

0. Review Chapter 2: All Concept Check (skip 12c) and True-False.
   Review Chapter 3: Concept Check 2,5bd,8,(skip 12c) and all True-False.
   Review Chapter 4: Concept Check 1a,2,3,4,5,6 and all True-False.
   Review Chapter 5: Concept Check 2,5.

1. (a) Indefinite integrals. Review Chapter 4: 6,14,16,21,22,28,29,30
   (b) Definite integrals. Review Chapter 4: 39,44,47,48,50
2. Find the derivatives of the following functions. Simplify your answer.
   (a) $g(s) = \sqrt{s} + \dfrac{1}{\sqrt[3]{s^4}}$   (b) $f(x) = \sin(\pi x) \displaystyle\int_{x^2}^{2} \sqrt{t^2+1}\, dt$
   (c) $g(s) = \displaystyle\int_{2s}^{3s} \frac{u^2-1}{u^2+1}\, du$   (d) $P(R) = \dfrac{E^2 R}{(R+r)^2}$, where $E, r$ are constants
3. If $f$ and $g$ are differentiable, find
   (a) $\dfrac{d}{dx}\left[\sqrt{f(x)}\right]$   (b) $\dfrac{d}{dx}\left[f(\sqrt{x})\right]$   (c) $\dfrac{d}{dx}\left[\sqrt{x} f(x)\right]$
   (d) $\dfrac{d}{dx}\left[f(g(x))\right]$   (e) $\dfrac{d}{dx}\left[f(f(x))\right]$   (f) $\dfrac{d}{dx}\left[\sqrt{\dfrac{f(x)}{g(x)}}\right]$
4. For the function $f(x) = |x^2 + x|$,
   (a) Sketch the graph of $f$. Use it to sketch the graph of $f'(x)$
   (b) Find a formula for $f'(x)$. Confirm that it is consistent with your graph in (a).
   (c) Find $\int_{-1}^{2} f(x)\, dx$

4B. §3.9: 46,47

5. Chapter 2 Review, # 50 (tangent/normal lines, implicit)
6. Sketch the graphs of the following functions. Clearly label axes, intercepts, local max/min.
   (a) $f(x) = \tan(\pi x)$   (b) $g(s) = 2\sin(3s)$   (c) $h(t) = 1 - \cos t$   (d) $h(t) = \sec t$
   (e) $f(x) = x^3 - x$   (f) $g(x) = x^2 + 4x + 2$   (g) $h(x) = x(x+1)^2(2-x)^3$
7. Mathematical definitions that require the limit concept.
   (a) State the definition of continuity of a function $f(x)$ at $x = a$.
   (b) State the definition of differentiability of a function $f(x)$ at $x = a$.
   (c) State the definition of the definite integral of a function $f(x)$ over the interval $x \in [a, b]$.
8. State both parts of the Fundamental Theorem of Calculus. Give an example of how you use each of them.
8. Use the definition of the derivative to find $f'(x)$ where $f(x) = 1/\sqrt{x+2}$.

9. §2.7: # 28 (vibrating string)

10. Use linearization to approximate the volume of a cylindrical shell of average radius $r$, height $h$, and thickness $\Delta r$.

11. Use linear approximations to estimate the amount of paint needed to apply a coat of paint 0.05cm thick to a hemispherical dome with diameter 50 m.

12. A particle moves on a vertical line so that its coordinate at time $t$ is $y = t^3 - 12t + 3, t \geq 0$.
    (a) Find the distance that the particle travels in the time interval $0 \leq t \leq 3$.
    (b) Find the particle's total displacement in the time interval $0 \leq t \leq 3$.

13. A particle moves on a line with velocity $v(t) = 3t^2 - 3t - 6$, $t \geq 0$.
    (a) Find the distance that the particle travels in the time interval $0 \leq t \leq 3$.
    (b) Find the particle's total displacement in the time interval $0 \leq t \leq 3$.

14. Find the local and absolute extreme values of the function on the given interval

$$f(x) = 10 + 27x - x^3 \ , \quad [0, 4]$$

15. Let $f(x) = \dfrac{4 - x}{3 + x}$.
    (a) Find the intercepts of $f$ and all its asymptotes. Also find the limiting behaviour of $f$ near its vertical asymptotes ($\lim_{x \to a^{\pm}} f(x)$).
    (b) Use the above information to sketch a guess for the graph of $f$.
    (c) Confirm details of your guess by finding the intervals where the function is increasing/decreasing and concave up/down.
    (d) Use algebra (long division in this case) to rewrite the function as a polynomial plus a proper quotient. Now sketch a graph of $f$ using translations/dilations of $y = \frac{1}{x}$. Which way of obtaining the sketch do you prefer?

16. The velocity of a wave of length $L$ in deep water is

$$v = K\sqrt{\frac{L}{C} + \frac{C}{L}}$$

    where $K$ and $C$ are positive constants. What is the length of the wave that gives the minimum velocity?

17. (a) Solve the differential equation $f'(x) = 8x - 3\sec^2 x$
    (b) Solve the initial value problem $f'(u) = \dfrac{u^2 + \sqrt{u}}{u}$ , $\quad f(1) = 3$.

18. Evaluate $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{x_i}\Delta x$, where $\{x_i\}$ is a partition of $[0, 1]$. (Hint: write as an integral.)

19. Evaluate $\int_0^2 3x - 2\sqrt{4 - x^2}\, dx$ .

20. (a) What is the difference between $\int f(x)\, dx$, $\int_a^x f(t)\, dt$, and $\int_a^b f(t)\, dt$.
    (b) What is the difference between $\frac{d}{dx}\left[\int_a^x f(t)\, dt\right]$ and $\frac{d}{dx}\left[\int_b^x f(t)\, dt\right]$, where $a \neq b$, if any?

21. Short answer questions.

(a) A rabbit population starts with 80 rabbits and increases at a rate on $dP/dt$ rabbits per week. What does

$$80 + \int_0^{12} \frac{dP}{dt}(t)\,dt$$

represent?

(b) If the units of $x$ are feet and the units for $a(x)$ are pounds per foot, what are the units for $da/dx$? what units does $\int_2^8 a(x)\,dx$ have?

(c) If $Q = Q(p)$ is the quantity (in pounds) of a ground coffee that is sold by a coffee compay at a price of p dollars per pound,
(i) What is the meaning of $Q'(8)$? What are its units?
(ii) Is $Q'(8)$ positive or negative? Explain.

(d) Evaluate
(i) $\displaystyle\int_0^1 \frac{d}{dx}\left[\frac{1}{1+x^2}\right] dx$
(ii) $\displaystyle\frac{d}{dx}\left[\int_0^1 \frac{1}{1+x^2}\,dx\right]$
(iii) $\displaystyle\frac{d}{dx}\left[\int_0^x \frac{1}{1+t^2}\,dt\right]$

22. Given the equation $\quad x\sin(\pi x) = \displaystyle\int_0^{x^2} f(t)\,dt\ ,\quad$ find $f(4)$.

23. Find the volumes of the solids obtained by rotating the region bounded by the curves $y = x$ and $y = x^2$ about the following lines:
(a) the $x$-axis (b) the $y$-axis (c) $y = 2$

24. (a) Use calculus to find the volume of a sphere of radius $R$.
(b) Find the volume of the sphere after a hole of radius $R/2$ has been drilled through its center.
(c) What percentage of the volume of the whole sphere is left after drilling the hole?

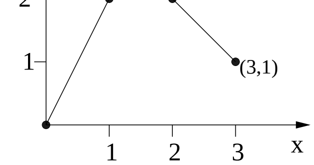
25. §5.5: 18 (avge vs maximal blood flow)

26. The reaction rate $V$ of a common enzyme reaction is given in terms of substrate level $S$ by

$$V = \frac{V_* S}{K+S}\ , \qquad S \geq 0$$

where $V_*$ and $K$ are positive constants.
Show that $V$ is an increasing function of $S$. Explain why it follows that $V$ has no absolute maximum value.
What is $\lim_{S\to\infty} V$ ?
Determine the concavity of the graph of $V$ (where is it concave up? where concave down?).
Sketch a graph of $V$ as a function of $S$.

27. (a) Find a function $f$ such that $F(x) = \int_0^x f(t)\,dt$ for the function $F$ whose graph is shown in the figure at right.
(b) What is the average of $f$ over [0,3]?




(c) What is the average of $F$ over [0,3]?

28. The dosage $D$ of diphenhydramine for a dog of body mass $w$ kg, is $D = kw^{2/3}$ mg, where $k$ is a constant. A cocker spaniel has mass $w = 10$ kg according to a veterinarians scale. Use linear approximations to estimate the maximum allowable error $\Delta w$ in $w$ if the percentage error $\Delta D/D$ in the dosage $D$ must be less than 5%.

29. page 340: #48.

30. §4.4: 63 (given graph of rate of change of volume of water in tank, find amount of water)