Math 2531: Review of Prerequisites

R1. Diagnostic Test A (Stewart, p xxvi): 1,2,4,8(e,g),10

R2. Sketch a graph of the following polynomials, solely using roots, symmetry, degree and sign of the leading coefficient.

(a) $f(x) = (x^2-1)^3$

(b) $f(x) = x^3 - cx$, $c > 0$

(c) $f(t) = 10t - 1.86t^2$

(d) $v(r) = r_0 r^2 - r^3$, $r_0 > 0$

(e) $F(r) = GMr/R^3$, $G, M, R > 0$

(f) $F(R) = GMr/R$, $G, M, r > 0$

R3. Find the derivatives of the following functions.

(a) $f(x) = \sin(x^3)$

(b) $f(t) = \dfrac{1}{1+2t^2}$

(c) $F(x) = \displaystyle\int_0^x \frac{\sin t}{t}\, dt.$

(d) $g(t) = \dfrac{1}{(1+t)^2}$

(e) $y = x^2 \sin(\pi x)$

(f) $F(R) = GMr/R$

(g) $y = \displaystyle\int_2^x \frac{t}{1+t^3}\, dt$

(h) $y = \displaystyle\int_1^{x^4} \cos(t^2)\, dt$

R4. Evaluate the following definite and indefinite integrals.

(a) $\displaystyle\int \frac{x}{\sqrt{1+x^2}}\, dx.$

(b) $\displaystyle\int \frac{dy}{y^3}$

(c) $\displaystyle\int x(2x+5)^8\, dx$

(d) $\displaystyle\int \frac{1-x^2}{x^2}\, dx$

(e) $\displaystyle\int \frac{x}{1-x^2}\, dx$,

(f) $\displaystyle\int \frac{x^3}{1-x^2}\, dx$

(g) $\displaystyle\int \frac{1}{1-x^2}\, dx$,

(h) $\displaystyle\int \cos(2x) + \sec^2(2x)\, dx$

(i) $\displaystyle\int_0^{\pi/8} \sec(2\theta)\tan(2\theta)\, d\theta$

(j) $\displaystyle\int \tan^2\theta \sec^2\theta\, d\theta$

(k) $\displaystyle\int \frac{t}{t^4+2}\, dt$

(l) $\displaystyle\int_{-1}^{1} \frac{\sin x}{1+x^2}\, dx$

(m) $\displaystyle\int \frac{d}{dx}\left[\frac{2x}{\cos^2 x + a^2}\right] dx$, a is a constant

(n) $\displaystyle\int v e^{v/3}\, dv$

(o) $\displaystyle\int_0^{1/2} \frac{dx}{2-3x}$

(p) $\displaystyle\int_0^{4/3} \frac{dx}{2-3x}$

(q) $\displaystyle\int \frac{x-1}{x^2-4x+5}\, dx$

R5. (a) Find the absolute max and min values of $f(x) = (x^2-1)^3$ on $[-1, 2]$.

(b) Using solely the roots, symmetry, degree and sign of leading coefficient, sketch a graph of the function and confirm that your answer in (a) makes sense.

R6. For what a, $0 \le a \le \pi$, does the function $f(x) = \sin(x) - \cos(x)$ have the greatest average over the interval $[a, a+\pi]$?

R7. (a) Write down the linear approximation of a function $f(x)$ at $x = a$.

(b) Write down the first 5 terms of the Taylor series expansion of $f(x)$ at $x = a$.

(c) Find the quadratic Taylor polynomial $p_2(x)$ that approximates $f(x) = \sqrt{1+x}$ near $a = 0$. Use it to approximate $\sqrt{1.1}$, $\sqrt{0.9}$ and compare with the more accurate value your calculator gives.