B. Approximating Integrals

Approximate double integrals (using definition) by

$$\iint_D f(x,y)\,dA \approx \sum_{k=1}^{n} f(x_k,y_k)\Delta A_k$$

where D has been partitioned into n subrectangles with area ΔA_k and (x_k,y_k) is a point in the kth subrectangle. Approximate triple integrals (using definition) by

$$\iiint_E f(x,y,z)\,dV \approx \sum_{k=1}^{n} f(x_k,y_k,z_k)\Delta V_k$$

where E has been partitioned into n sub-boxes with volume ΔV_k and (x_k,y_k,z_k) is a point in the kth sub-box.

C. Double and Triple Integrals

Set up and evaluate the integrals in cartesian, polar/cylindrical or spherical coordinates
Always sketch the region of integration
Change the oder of integration, if necessary to simplify integration
Change from cartesion to cylindrical or spherical, if necessary to simplify integration
Applications: computing areas, volumes, masses, centroids.

STUDY PROBLEMS

2. Review Chapter 14: 63 (Note: minimizing a distance is equivalent to minimizing the distance squared.)

B. Approximating Integrals

1. Chapter 15 Review, Exercise 2
2. Approximate $\int_0^1\int_0^1\int_0^1 xzy^2\,dxdydz$ using
   (a) 1 sub-box, midpoint in each box
   (b) 8 sub-box, midpoint in each box
   (c) compare your answer with the exact value

C. Double and Triple Integrals

1. Chapter 15 Review, Exercises: 9,10,25,34,42,44
2. The integral

$$\iint_R (9-x^2)\,dA$$

where $R = [0,2] \times [0,4]$, represents the volume of a solid. Sketch that solid.

3. Evaluate the integral $\int_0^1 \int_y^1 \cos(x^2)\,dxdy$
4. Find the volume bounded by $y = x^2 + z^2$ and $y = 3$.
5. Find the volume above $z = \sqrt{3x^2 + 3y^2}$ and below $x^2 + y^2 + z^2 = 4$.
6. Set up the integral $\int\int\int_T y\,dV$ where T is the tetrahedon bounded by $x = 0$, $y = 0$, $z = 0$ and $2x + y + z = 2$ in the 3 different orders $dV = dzdydx$, $dV = dxdydz$, $dV = dydxdz$.
7. Find the centroid of the hemisphere of radius a given by $x^2 + y^2 + z^2 = a^2$, $z \geq 0$.
8. Find the volume of the region bounded by $y^2 + z^2 = 9$, $x = 0$, $y = 3x$, $z = 0$.
9. Evaluate the integral $\int\int\int_E z\,dV$ where E is bounded by the $y = 0$, $z = 0$, $x = 0$, $x + y = 2$, $y^2 + z^2 = 1$
10. Evaluate the integral $\int\int\int yz\,dV$ where E lies above the plane $z = 0$, below the plane $z = y$, and inside the cylinder $x^2 + y^2 = 4$.
11. (a) Set up the integral $\int\int\int_E x^2 + y^2 + z^2\,dV$ where E is the region bounded below by the cone $\phi = \pi/6$ and above by the sphere $\rho = 2$ (i) in cartesian coordinates, (ii) in cylindrical coordinates, (iii) in spherical coordinates.
    (b) Evaluate the integral.
12. Consider $\int_0^{\pi/2} \int_0^2 \int_0^{\sqrt{3}} r(1 - r^2)\,dz\,dr\,d\theta$. Sketch the region of integration. Write the integral in cartesian and in spherical coordinates. Evaluate the integral.
13. Find the volume of the intersection of the two cylinders $x^2 + y^2 = r^2$ and $x^2 + z^2 = r^2$.