1. Solutions of first order ODEs $\frac{dy}{dt} = f(t, y)$
   ○ Show that a given function solves a given ODE
     Examples: HW 1: 3
   ○ Solve separable equations
     
     $y' = f(t)g(y)$
     
     using separation of variables. Careful!! Only separate variables if $g(y) \neq 0$. Also: be careful with your algebra, and always check your antiderivatives.
     Examples: 11th ed: §2.2 7,8,12,13. HW 1: 4. HW 2: 5,6,8. HW 3: 3,4,6,8,9
   ○ Solve linear equations
     
     $y' + p(t)y = q(t)$
     
     using integrating factors.
     What is the main idea behind the method? Can you derive the integrating factor without memorizing formulas?
   ○ Investigate autonomous equations drawing the phase line and direction fields. Find equilibria and their stability.
     Examples: §1.1, §2.5: first set of problems. HW 2: 1-3,5. HW 3: 5,9
   ○ Know: Integration by parts, partial fraction, substitution.
   ○ Use the solutions you found (as well as direction fields and phaseline where available) to investigate limiting behaviour as $t \to \infty$, or as $t \to 0$, and the dependence of the limiting behaviour on the initial condition.
     Examples: §2.1: Example 3,4. HW 2: 1-3,7. HW 3: 5,7,8,9,10

2. Mathematical Models
   ○ Find differential equation models for simple applications (mixing, population dynamics, falling objects with or without drag, raindrop, bucket)
     Examples: 10th ed: §1.1: 22,23,24,25. HW 2: 4-8, HW 3: 4-10

3. Theory
   ○ What can you say about the existence and uniqueness of a solution to a linear first order ode, $y' + p(t)y = q(t)$?
   ○ What can you say about the existence and uniqueness of a solution to a nonlinear first order ode, $y' = f(t, y)$?
     Examples: HW 2: 1-3,9,10. HW 3: 1-3,11
   ○ Give an example of a nonlinear ODE whose solution blows up in finite time.
   ○ Give an example of a linear ODE whose solution does not exist for all time.

4. Combo problem
   Consider the temperature $T$ of coffee in a cup in a room of ambient temperature $T_a$.
   (a) Using Newton’s law of cooling, find an (autonomous) ODE modeling the temperature $T$.
   (b) Draw the phaseline, determine the stability of all equilibria, and draw several solution curves in the $t$-$T$ plane.
   (c) Find all the solutions to the ODE with initial condition $T(t_0) = T_0$, using the method of separation of variables.
   (d) Find all the solutions to the ODE with initial condition $T(t_0) = T_0$, using the method of integrating factor.
   (e) For how long does the temperatures function $T(t)$ exist, continuously? Explain, using both your above results and the theoretical results of §2.4.
   (f) If the coffee in two cups in the same room have two different temperatures at time $t_0$, will they ever have the same temperature? Explain.