HOMEWORK DAY 3 – *Separation of Variables. §1.3*

1. Find the general solution to the following DEs

   (a) $y' = -2ty\ , \quad y \neq 0$

   (b) $x' = -x/t\ , \quad x \neq 0, \ \ t > 0$

(c) $(60 - t)x' = 2x \ , \quad x \neq 0, \quad t > 60$

(d) $u' = 5 - 2u \ , \quad u \neq 5/2$

(e) $t\mu' = (1+t)\mu$, $\quad \mu > 0, \quad t > 0$

(f) $y' + y + \frac{1}{y} = 0$, $\quad y \neq 0$

2. Solve the given IVPs

   (a) $\dfrac{dx}{dt} = tx^3 \ , \quad x(0) = 1$

   (b) $\dfrac{dx}{dt} = tx^3 \ , \quad x(0) = 0$

   (c) $\dfrac{dy}{dx} = k - ry \ , \quad y(0) = k/(2r)$ where k, r constants

(d) $\dfrac{dy}{dt} = 0.2(1 - \dfrac{y}{5})y\ , \quad y(0) = 10$

HOMEWORK DAY 4 – *Linear 1st order: Integrating factors. §1.4.1*

3. page 41: 1.

4. page 42: 2a.

5. page 42: 2c.

6. page 42: 3d.

7. page 42: 3f. (assume $t > 0$)

8. page 42: 4.

9. page 42: 5.

10. Newton's law of cooling states that the temperature of an object changes at a rate proportional to the difference between the temperature of the object an the temperature of its surroundings. Suppose H is the temperature of a hot object placed into a room whose temperature is 70 degrees, and t represents time, and $k > 0$ is a positive parameter. Which of the following differential equations best models the temperature $H(t)$?

    ☐ $dH/dt = -kH$ ☐ $dH/dt = k(H - 70)$ ☐ $dH/dt = -k(H - 70)$
    ☐ $dH/dt = -k(70 - H)$ ☐ $dH/dt = -kH(H - 70)$

    Give 2 short and concise reasons for your choice.

11. page 34: 5.

12. page 32: 3. Use the solution to the cooling law $dT/dt = -k(T - T_e)$, $T(0) = T_0$ derived in class, $T(t) = T_e + (T_o - T_e)e^{-kt}$.

13. page 46: 1.

14. page 46: 4. You may use the formula $\int e^{at}\cos bt\,dt = \frac{e^{at}}{a^2+b^2}(a\cos(bt) + b\sin(bt))$. (Dont plot the function and find the time lag. They will be shown in the solutions.)

Extra Credit: Derive the formula given above using complex variables
$\cos bt = \frac{1}{2}(e^{ibt} + e^{-ibt})$, $\sin bt = \frac{1}{2i}(e^{ibt} - e^{-ibt})$.