HOMEWORK DAY 15 – *Applications. §2.2.3, 2.2.4*

1. A given spring is governed by $x''+x'+x=0$. Classify the system as underdamped, critically damped, overdamped, or not damped. Sketch the shape of the solution $x(t)$.

2. Repeat for $4x''+25x=0$.

3. p.99: 3.

4. A spring is govered by $x'' + 3\gamma x' + 2x = 0$. For which values of γ is the spring underdamped, critically damped, overdamped, not damped?

5. An oscillator with damping is governed by $x'' + 3\delta x' + \kappa x = 0$, where δ, κ are positive parameters. Plot the set of point in the δ-κ plane where the system is (i) critically damped, (ii) overdamped, (iii) underdamped. (Show all regions in one plot)

6. p.99: 4. Assume $L > 0$ ($I(t)$ denotes the current at time t.)

7. Describe the possible types of solutions of $LI'' + I' + I = 0$ if $L \leq 0$.

8. The solution $x(t)$ of the following initial value problems is given.
Write the solution in the form

$$x(t) = A\cos(\beta t - \phi) = A\cos(\beta(t - \phi/\beta))$$

Sketch a graph of $x(t)$ that captures the initial condition, the period, amplitude and phase shift.

(a) $x'' + 4x = 0\ ,\quad x(0) = 3\ ,\quad x'(0) = 8$. Solution $x(t) = 3\cos(2t) + 4\sin(2t)$.

(b) $x'' + x = 0\ ,\quad x(0) = -1\ ,\quad x'(0) = \sqrt{3}$. Solution $x(t) = -\cos(t) + \sqrt{3}\sin(t)$.

9. The solution $x(t)$ of the following initial value problems is given.
Write the solution in the form

$$x(t) = Ae^{\alpha t}\cos(\beta t - \phi)$$

Sketch a graph of $x(t)$ that captures the initial condition, quasiperiod, and envelope. The graph does not have to show the phase shift, as this is often difficult to see.

(a) $x'' + 4x' + 20x = 0$, $x(0) = 3$, $x'(0) = -2$. Solution: $x(t) = e^{-2t}(\sin(4t) + 3\cos(4t))$.

(b) $x'' - x' + \frac{5}{2}x = 0$, $x(0) = 3$, $x'(0) = 1$. Solution: $x(t) = e^{t/2}(3\cos(\frac{3}{2}t) - \frac{1}{3}\sin(\frac{3}{2}t))$

10. p.99: 1. Include sketch of graph of $x(t)$.

11. p.110: 1(a)

(b)

(c)

(d)

(e)

(f)

(h)

(j)

12. p.111: 2a

13. p.111: 2b

14. p.111: 2c

15. p.111: 5

16. A spring-mass system has a spring constant of 3 N/m. A mass of 2 kg is attached to the spring, and the motion takes place in a viscous fluid that offers a resistance numerically equal to the magnitude of the instantaneous velocity ($\gamma = 1$). If the system is driven by an external force of $(3\cos 3t - 2\sin 3t)N$ determine the steady state response. Express your answer in the form $R\cos(\omega t - \phi)$.

HOMEWORK DAY 17 – *Resonance and beats §2.3.2*

17. p.114: 3

18. p.114: 5

19. A mass of 400g stretches a spring 20 cm. The mass is given a positive displacement of 4 cm from this equilibrium position and released with no initial velocity. Assume that there is no damping and that the mass is acted on by an external force of $2\cos 3t$ N.

    (a) Formulate the initial value poblem describing the motion of the mass.

    (b) If the external force is replaced by a force $4\sin\omega t$ of frequency ω, find the value of ω for which resonance occurs.

20. *Sums of trig functions of unequal period.* The following relations let us rewrite sums of sines and cosines with *unequal* periods,

$$\cos(a) + \cos(b) = 2\cos(\frac{a+b}{2})\cos(\frac{a-b}{2})$$

$$\cos(a) - \cos(b) = -2\sin(\frac{a+b}{2})\sin(\frac{a-b}{2})$$

$$\sin(a) + \sin(b) = 2\sin(\frac{a+b}{2})\cos(\frac{a-b}{2})$$

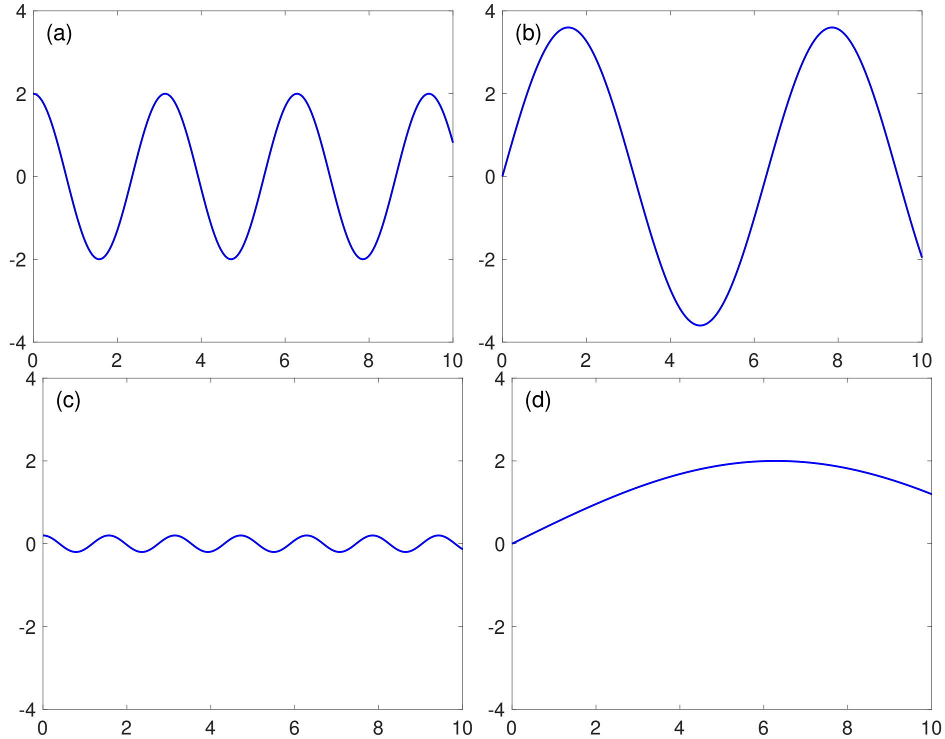
(a) Given these relations, write the function $f(t) = \cos(17t) - \cos(15t)$ as a product of two trigonometric functions.

(b) Sketch a graph of it, clearly indicating the envelope and the period of the envelope.

(c) Can you explain why this function is referred to as a **beat**?

Below: Short answer questions. No need to justify, but be able to explain if asked for!

21. The functions below are solutions of $x'' + ax = 0$ for different values of $a > 0$. Which case corresponds to the largest value of a?



22. Consider the harmonic oscillator described by $3y'' + 4y' + y = 0$.

    (a) Classify this oscillator as underdamped, overdamped, critically damped, not damped.

    (b) Does this oscillator oscillate?

23. The position of a certain spring-mass system satisfies the initial value problem

$$2x'' + \gamma x' + x = 0 \ , \quad x(0) = 2, \quad x'(0) = 0$$

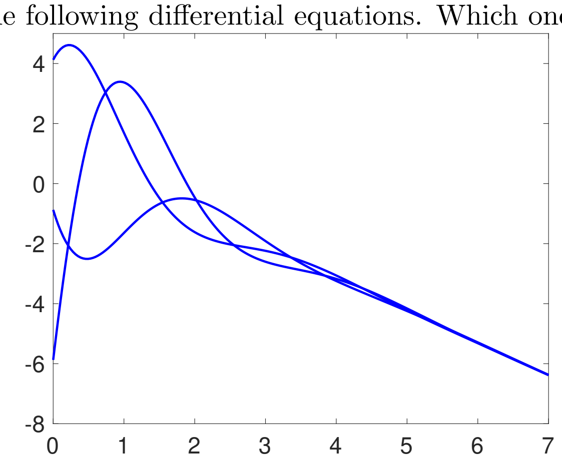
For which values of the coefficient γ is the system not damped?

For which values of the coefficient γ is the system underdamped?

critically damped?

overdamped?

24. The functions plotted below are solutions to one of the following differential equations. Which one?



Is a positive or negative?

(a) $x'' + ax' + 4x = 3 - 3t$

(b) $x'' + ax' + 4x = 3e^{2t}$

(c) $x'' + ax' + 4x = \sin \frac{2\pi t}{9}$

(d) $x'' + ax' + 4x = 2t^2 + 3t - 4$

25. Which of the following forced 2nd order equations has solutions exhibiting *resonance*?

(a) $x'' + x = \cos t$

(b) $x'' + x = 5\cos t$

(c) $x'' + x = -10\cos t$

(d) All of the above

(e) None of the above

26. Which of the following forced 2nd order equations has solutions exhibiting *beats*?

(a) $x'' + 3x = 10\cos(2t)$

(b) $x'' + x = 5\cos(2t)$

(c) $x'' + 9x = \cos(3t)$

(d) All of the above

(e) None of the above

27. The differential equations $x'' + 100x = 2\cos(\omega t)$ has solutions displaying *resonance* when

(a) $\omega = 10,000$

(b) $\omega = 10$

(c) $\omega = 9$

(d) All of the above

(e) None of the above