Math 375 Review Exam 1

Below I list the main topics we covered so far, questions you should know how to answer, and specific skills you should have for the exam. Good related problems are those in HW 1 through HW 4.

- **Taylor series**
  - Write the Taylor series for a function $f(a+h)$ near the basepoint a, in the form of
    $$f(a+h) = p_n(h) + R_n$$
    where $p_n(h)$ is the Taylor polynomial of degree n and R_n is an expression for the remainder (the error in approximating f by p_n).
  - An approximation $g(h)$ of $f(h)$ is of order p if $g - f = O(h^p)$. Determine the order p of an approximation by plotting $|g - f|$ on a log-log scale.
  - Given plots of $|g - f|$ on a log-log scale, explain the slopes you observe, referring to Taylor series if needed.
  - Use Taylor series to prove convergence and derive the order of iterative schemes (fixed point iterations, Newton's method).
  - Use Taylor series to derive the order of a finite difference approximation of a derivative.
- **Numerical methods**
  - What methods did we discuss to solve $f(x) = 0$? For each method
    - give a formula for the solution at the kth step
    - give a picture of the sequence of solutions, if possible (such as for FPI, and Newton's)
    - what is the order of the method, under which conditions?
    - given the error at the kth step, can you approximate the error at the $k+1$st step?
  - Estimate the number of steps Bisection Method requires to obtain the solution to within p digits of precision.
- **Algorithms and Matlab**
  - write down algorithms that implement the methods we discussed to solve nonlinear equations
  - write down matlab code for those algorithms
  - State results of a given matlab code.
- **Accuracy**
  - What is machine precision?

– How can accuracy be lost in a numerical simulation? Give specific examples.

– If an approximation error is $O(h^p)$, by how much does the error decay each time h is halved?